*JKAU: Eng. Sci.*, Vol. **31** No. 1, pp: 3 – 18 (1441 A.H. / 2020 A.D.)

Doi: 10.4197/Eng. 31-1.1

# Representations of a Coherent Reliability System via Signal Flow Graphs

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*Abstract.* A coherent reliability system (CRS) is one that is causal, monotone and with relevant components. We restrict ourselves herein to the case of a two-state system with statistically independent two-state components. One of the most prominent methods to study the reliability of such a system is to characterize it via recursive relations together with boundary conditions. This paper presents recursive relations as well as boundary conditions for six entities pertaining to a CRS. These are (a) expressions of monoform literals for either system success or failure (b) probability-ready expressions for either system success or failure, and (c) all-additive formulas for either system reliability or unreliability. Each of the six entities considered is represented by an acyclic (loopless) Mason signal flow graph (SFG). The SFG for system success or failure is isomorphic to a Reduced Ordered Binary Decision Diagram (ROBDD) which is the optimal data structure for a Boolean function. The interrelations between the SFGs demonstrate optimal procedures for implementing (a) the probability (real) transform of a Boolean function, (b) inversion or complementation of a Boolean function, and (c) disjointing or orthogonalization of a sum-of-products expression of a Boolean function. The SFGs discussed herein reduce to elegant symmetric graphs for the special cases of a partially-redundant system (k-out-of-n system) and a threshold system (weighted k-out-of-n system). The results obtained suggest a renaissance of the use of signal flow graphs in the study of system reliability for both coherent and noncoherent systems and for particular classes thereof.

*Keywords*: Coherent reliability system, Recursive relation, Boundary condition, Signal flow graph.

## 1. Introduction

This paper is intended for a revival of the utilization of signal flow graphs (SFGs) in the investigation of system reliability. A few papers handled this topic decades ago [1-7] and a very recent paper [8] dealt with it for the restricted class of 2-state coherent threshold systems of s-independent components. Our current paper extends the scope of the work in [8] by exploring SFG utility in a wider class of reliability systems, namely general coherent 2-state systems of statistically independent 2-state components. The extension of this representation to multi-state systems of multi-state components is promising, indeed, but will be deferred to future work.

This paper reviews recursive relations together with boundary conditions for six quantities characterizing a Coherent Reliability System (CRS). These are (a) unate expressions for system success and failure, with uncomplemented and complemented variables, respectively, and (b) probability-ready expressions for both system success and failure, and (c) all-additive formulas for system reliability and unreliability. Each of the six quantities treated herein is modeled by a loopless (acyclic) signal flow graph (SFG).

The SFG for system success or failure is isomorphic to a Reduced Ordered Binary Decision Diagram (ROBDD) [9-13], which has been developed to serve as an efficient data structure for switching functions. The interrelations between the SFG's demonstrate efficient procedures realizing (a) the real (probability) transform of a switching function, (b) complementation (inversion) of a switching function, and (c) orthogonalization (disjointness) of a sop expression of a switching function. The SFGs discussed herein for a CRS reduce to regular graphs for the special case of a k-out-of-n system (partially-redundant system) [11-18]. They also reduce to elegant graphs for the non-symmetric cases of a threshold system (weighted k-out-of-n system) [8, 19, 20], a double-threshold system [21], or a k-to-*l*-out-of-n system [22, 23].

Work in this paper is a natural extension for our earlier work dealing with coherent threshold systems [8]. However, the systems considered herein are only coherent and not necessarily threshold. This means that the SFGs obtained in the current case for system success or failure cannot be immediately used to enumerate minimal pathsets or minimal cutsets, by directly computing the complete sum for system success or failure. However, such an enumeration is still possible, albeit with an extra step. The SFG for a coherent system success or failure directly produces a general syllogistic formula for success or failure, which is not necessarily absorptive and has to be converted into an absorptive form by absorbing any term that subsumes another. The resulting absorptive formula is a canonical one representing the complete sum for success (disjunction of all minimal pathset) or the complete sum for failure (disjunction of all minimal cutsets).

The organization of the remainder of this paper is as follows. Section 2 presents some useful nomenclature necessary for understanding the rest of the paper. Section 3 points out the existence of useful expansions for general switching (Boolean) functions, and stresses the utility of these expansions for monotonically non-decreasing and monotonically non-increasing switching functions. Section 3 then develops the aforementioned expansions to ones for the success, failure, reliability, and unreliability of a CRS and translates them (together with boundary conditions) into appropriate Signal Flow Graphs (SFGs). This is followed by a detailed discussion in Sec. 4 on the merits and interrelations of the SFGs presented in Section 3. Section 5 adds a few concluding remarks.

## 2. Nomenclature

### *2.1 Coherent Reliability System (CRS)*

A CRS is a reliability system characterized by three features in the Boolean domain concerning its success S as a function S(**X**) of component successes **X** [8, 12, 20]

(a) causal

$$\mathbf{S(0) = 0, S(1) = 1.} \tag{1}$$

(b) monotonically increasing, *i.e.*,

$$\mathbf{\{X \geq Y\} \text{ implies } \{S(X) \geq S(Y)\}.} \tag{2}$$

(c) of relevant (non-dummy) components, *i.e.*,

$$\mathbf{\frac{\partial S}{\partial X_i} = S(X \mid X_i{=}0) \oplus S(X \mid X_i{=}1),} \tag{3}$$

is not identically 0.

### *2.2 Probability-Ready Expression (PRE)*

A PRE is a switching formula that can be directly converted, on a one-to-one basis, to a probability expression called the *probability* or real *transform* [12, 24-28]. In a probability-ready formula

(a) Any sum-of-products (sop) sub-formula has products that are mutually exclusive (disjoint or non-overlapping).

(b) Any product-of-sums (pos) sub-formula has statistically-independent sums.

The transition from a PRE to a probability formula is attained by replacing switching variables $X_i$ and $\overline{X}_i$ by the probabilities of their being equal to 1, *i.e.*, by

$$\boldsymbol{p_i \equiv \Pr\{X_i = 1\} = E\{X_i\}, q_i \equiv \Pr\{\overline{X}_i = 1\} = E\{\overline{X}_i\} = 1 - p_i},$$

and substituting arithmetic addition and multiplication for their logical counterparts (disjunction and conjunction operations).

### *2.3 Linear Signal Flow Graph*

A linear signal flow graph (SFG) [29-42], is a specialized directed graph whose nodes represent certain variables, and whose branches represent transmittances between pairs of nodes. A branch outgoing from a certain node and incident on a (not necessarily different) node adds to the value of the latter node the value of the former node weighted (multiplied) by the transmittance carried by this branch. There are two main closely-related types of an SFG [34], namely Mason SFG [29], and Coates SFG [30]. We confine ourselves herein to the SFG type that is prominent in Electrical Engineering applications, namely the Mason SFG. This is an SFG in which the value of any specified non-source node equals the weighted sum of nodes that influence the specified node (*i.e.*, the sum of the values of the influencing nodes, each multiplied by the transmittance on the edge originating at the influencing node and incident on the specified node). Good tutorial expositions on SFG's are available in textbooks on automatic control such as [37].

### *2.4 The Complete Sum of a Boolean Function CS (f)*

The complete sum of a switching function is an ORing of all the prime implicants of the function, and nothing else [43, 44]. When the function f is the system success S, the prime implicants are called the minimal pathsets of the system [20, 45], and when f is the system failure $\overline{S}$, the prime implicants are called the minimal cutsets of the system [20, 45]. Work in this paper is confined to a coherent system, exemplified by source-to-terminal connectivity in a probabilistic network. In this case, the pathsets and cutsets have geometric as well as logical interpretations. Moreover, coherency dictates that the complete sum (for both system success and system failure) be the sole irredundant-disjunctive form of the pertinent function, and hence it coincides with its minimal sum. Coherence is also manifested in the condition that the prime implicants involve uncomplemented literals only for system success and complemented literals only for system failure.

### *2.5 A Syllogistic Formula for a Boolean Function*

A syllogistic formula for a switching function f is a possibly non-absorptive sop formula for the function, *i.e.*, it is a disjunction of products, none of which can be absorbed by (any disjunction of) other products in the formula. Therefore, a syllogistic formula includes all the prime implicants (and possibly some non-prime implicants) of the function [43, 44]. The compete sum is a special syllogistic formula that is both minimal and canonical.

## 3. Six SFGs for a Typical Coherent System

Figure 1 presents a 5-node 7-element source-to-terminal (st) network, taken from [45], which can be conveniently called a double-bridge network. This network serves as a typical example for a general coherent system. Table 1 lists the six quantities to be studied herein, which are

(1) $S_{minimal}$ = S = System success (in minimal form as the disjunction of all minimal pathsets),

(2) $\overline{S}_{minimal} = \overline{S} =$ System failure (in minimal form as the disjunction of all minimal cutsets),

(3) $S_{PRE}$ = System success in a probability-ready-form,

(4) $\overline{S}_{PRE}$ = System failure in a probability-ready-form,

(5) R = System reliability, obtained as $E\{S_{PRE}\}$ by replacing component successes/failures by their expectations and substituting arithmetic addition and multiplication for ORing and ANDing (on a one-to-one basis), and

(6) U = System unreliability, obtained as $E\{\overline{S}_{PRE}\}$ by replacing component successes/failures by their expectations and substituting arithmetic addition and multiplication for ORing and ANDing (on a one-to-one basis).

Rushdi and Alturki [8] based their SFG representation on the Boole-Shannon expansion of a Boolean function $f(\mathbf{X}) = f(X_1, X_2, \ldots, X_{i-1}, X_i, X_{i+1}, \ldots, X_n)$, namely

$$\mathbf{f(X) = \overline{X}_i\, f_0 \vee X_i\, f_1} \tag{4}$$

where $f_0$ and $f_1$ are restrictions, subfunctions, ratios, quotients, or cofactors of $f(\mathbf{X})$ given by

$$\mathbf{f_0 = f(X|0_i) = f(X_1, X_2, \ldots, X_{i-1}, 0, X_{i+1}, \ldots, X_n),} \tag{5}$$

$$\mathbf{f_1 = f(X|1_i) = f(X_1, X_2, \ldots, X_{i-1}, 1, X_{i+1}, \ldots, X_n),} \tag{6}$$

Note that equation (4) uses ANDing and ORing, which are usually designated as logical multiplication and addition.

Equations (4-6) can be transformed from the Boolean domain to the probability domain [8, 12] by taking the expectations of both sides of each equation. When f stands for system success, the result is

$$\mathbf{R = E\{S(X)\} = q_i\, R_0 + p_i\, R_1,} \tag{7}$$

$$\mathbf{R_1 = E\{S(X|0_i)\} = R(p_1, p_2, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_n),} \tag{8}$$

$$\mathbf{R_2 = E\{S(X|1_i)\} = R(p_1, p_2, \ldots, p_{i-1}, 1, p_{i+1}, \ldots, p_n),} \tag{9}$$

Now, we construct an SFG for each of the quantities defined in Table 1. In any of these SFGs, the value of each specified node is the weighted sum of nodes from which arrows incident on this specified node originate, where the weighting of any of these nodes is through multiplication with the transmittance on the edge emanating from it towards the specified node. We utilize the Boole-Shannon expansion in the Boolean domain (4) to construct SFGs for $S_{PRE}$ and $\overline{S}_{PRE}$ in Fig. 4 and 5, respectively, and likewise employ the same expansion in the probability domain (7) to construct SFGs for R and U in Fig. 6 and 7, respectively. In retrospect, we construct SFGs in Fig 2 and 3, respectively for S and $\overline{S}$, in monoform representation. Figure 2 is exactly Fig. 4, but with each complemented literal $\overline{X}_i$ being replaced by 1, while Fig. 3 is a replica of Fig. 5 with each uncomplemented literal $X_i$ being replaced by 1. While Fig. 4 and 5 contain a mixture of uncomplemented and complemented component literals, Fig. 2 has only uncomplemented literals and Fig. 3 has only complemented literals. Logical addition and multiplication (ORing and ANDing) are implicitly assumed in Fig. 2-5, while usual arithmetic addition and multiplication are assumed in Fig. 6 and 7. Other similarities and distinctions among Fig. 2-7 are noted below.

## 4. Features and Interrelations of the Six SFGs

The six SFGs in Fig. 2-7 are very similar with possible differences in their transmittances, nature of source nodes, and nature of addition/multiplication operations (logical or arithmetic). The six quantities represented by the SFGs in Fig. 2-7 are expressed by the formulas of Table 1. It can be easily verified that

$$\mathbf{S \wedge \overline{S} = 0,} \tag{10}$$

$$\mathbf{S \vee \overline{S} = 1,} \tag{11}$$

$$\mathbf{S_{PRE} = S,} \tag{12}$$

$$\mathbf{\overline{S}_{PRE} = \overline{S},} \tag{13}$$

$$\mathbf{R + U = 1.0,} \tag{14}$$

It should be noted that each of S and $\overline{S}$ is given in a complete–sum form, which (due to coherency) happens to be also a minimal-sum form. Neither of the two expressions for $S_{PRE}$ and $\overline{S}_{PRE}$ is in minimal form. If all complemented literals in $S_{PRE}$ and uncomplemented literals in $\overline{S}_{PRE}$ disappear (through being replaced by 1), the original formulas reduce to the non-absorptive syllogistic formulas for S and $\overline{S}$ that might be read from Fig. 2 and 3. A non-absorptive syllogistic formula for S contains non-minimal paths (non-prime implicants) besides the minimal paths (prime implicants). Similarly, a syllogistic formula for $\overline{S}$ contains non-minimal cutsets (non-prime implicants) as well as minimal cutsets (prime implicants). The complete sum for S (or $\overline{S}$) is an absorptive version of the resulting syllogistic formula, in which non-prime implicants are absorbed and only prime implicants are retained. The six SFGs in Fig. 2-7 are of beneficial pedagogical values. They provide immediate visual insight, and they constitute pictorial proofs for several important results that we explore in the following subsections.

### *4.1 The Functions S and $\overline{S}$ for a CRS System are Unate*

A function is called unate if it is possible to express it using only non-complemented literals or complemented ones. In the former case, the function is said to be a monotonically non-decreasing function in its variables, while in the latter case it is said to be a monotonically non-increasing function in its variables. The SFG in Fig. 2 is a pictorial proof that the success S of the CRS considered is a monotonically non-decreasing function in its arguments. Note that no edge transmittance in Fig. 2 is a complemented variable (Each edge transmittance is either 1 or $X_i$). Similarly, Fig. 3 is a pictorial proof that the failure $\overline{S}$ of the CRS considered is a monotonically non-increasing function in its arguments. This is due to the fact that no edge transmittance in Fig. 3 is an uncomplemented literal (each edge transmittance is either 1 or $\overline{X}_i$).

### *4.2 The Expressions for S and $\overline{S}$ Syllogistic Formulas*

Each of the expressions for S and $\overline{S}$ is a syllogistic formula (a sum-of-products formula that contains all the prime implicants of the pertinent function [43, 44]), but not necessarily an absorptive formula (one that has no term that can be absorbed by others [43]). Hence, each of the expressions ABS(S) and ABS($\overline{S}$) is a complete sum or a Blake Canonical Form (a disjunction of all prime implicants, and nothing else), where the symbol ABS(f) denotes an absorptive formula for f, *i.e.*, a formula in which any term absorbable in others is deleted. Since the complete sum and minimal sum are identical for a unate function [46], the expressions of ABS(S) and ABS($\overline{S}$) are also the minimal expressions for the unate functions S and $\overline{S}$.

### *4.3. The Expressions for S and $\overline{S}$ are Generally not Shellable*

A Boolean function in sop form is shellable if its terms can be disjointed without an increase in their number. Threshold Boolean functions are known to be shellable [8, 47]. However, general Boolean functions are not necessarily shellable. The present formulas for S and $\overline{S}$ are obviously not shellable, a fact associated with that of their being non absorptive, which results from the nature of their SFGs that lacks regularities possessed by the SFGs of threshold functions.

### *4.4. The SFGs of S and $\overline{S}$ are Essentially ROBDDs*

The Reduced Ordered Binary Decision Diagram (ROBDD) is a data structure for *general* switching functions, and has extensive applications in reliability [48, 49]. The SFGs for S in Fig. 2 and $\overline{S}$ in Fig. 3 are, in fact, implementations of the ROBDD for the class of unate switching functions. Apart from the unateness restriction, these SFGs have the same characteristics as the ROBDD [11-13, 49].

### *4.5 Efficient Inverse Algorithms*

Availability of complementary SFGs allows for a pedagogical understanding of many existing complementation or inversion procedures [50-52], both for the Boolean and probability domains. For example, one can start from the sink node of $S_{PRE}$ in Fig. 4, use expansion until the leaf nodes are reached, complement the value of each leaf (source) node (replace 0 by 1 and 1 by 0), thereby effectively transferring to Fig. 5, and finally go back to the sink of Fig. 5 which is $\overline{S}_{PRE}$. Other ways for complementation are possible. One can perform the converse operation of going from $\overline{S}_{PRE}$ to $S_{PRE}$ (expand in Fig. 5, complement leaves (sources), transfer to Fig. 4 and go to its sink). One can also achieve disjoint complementation by going from S (Fig. 2) to $\overline{S}_{PRE}$ (Figure 5). Complementation is also possible in the probability domain by going from R (Fig. 6) to U (Fig. 7) or vice versa.

## 5. Conclusions

A Coherent Reliability System (CRS) is the most prominent reliability model. Many of its features, probabilities, and algorithms are studied herein in terms of various recursive relations and boundary conditions, which are pictorially displayed in terms of various loopless Signal Flow Graphs. The success and failure of a CRS are shown to be unate Boolean functions whose minimal and complete sum expressions are identical. Interrelations among the SFGs demonstrate optimal procedures for mutual complementation among S and $\overline{S}$, for disjointing S and $\overline{S}$ to obtain PRE expressions $S_{PRE}$ and $\overline{S}_{PRE}$. The probability or real transforms of $S_{PRE}$ and $\overline{S}_{PRE}$ (namely, the reliability R and unreliability U) are obtained by replacing logical variables by their expectations and replacing ANDing and ORing operations by arithmetic multiplication and addition. The probability transforms of S and $\overline{S}$ are exactly the same as those of $S_{PRE}$ and $\overline{S}_{PRE}$ and can be obtained in a two-step fashion by first converting S and $\overline{S}$ to $S_{PRE}$ and $\overline{S}_{PRE}$ and then transforming them [52]. These transforms are also obtained directly via the conventional Inclusion-Exclusion principle or via a recursive version of it [45]. The graph complexity of each of the SFG's encountered herein is exponential in the worst case.

**Table 1. Algebraic expressions for six entities to be expressed via SFGs.**

| Case | Entity | Expression |
|---|---|---|
| 1 | $S_{minimal}$ | $X_2X_6 \vee X_1X_4X_7 \vee X_1X_3X_6 \vee X_2X_5X_7 \vee X_1X_3X_5X_7 \vee X_2X_3X_4X_7 \vee X_1X_4X_5X_6$ |
| 2 | $\overline{S}_{minimal}$ | $\overline{X}_1\overline{X}_2 \vee \overline{X}_6\overline{X}_7 \vee \overline{X}_2\overline{X}_3\overline{X}_4 \vee \overline{X}_1\overline{X}_3\overline{X}_5\overline{X}_6 \vee \overline{X}_2\overline{X}_3\overline{X}_5\overline{X}_7 \vee \overline{X}_4\overline{X}_5\overline{X}_6$ |
| 3 | $S_{PRE}$ | $X_2X_6 \vee X_2X_5\overline{X}_6X_7 \vee \overline{X}_1X_2X_3X_4\overline{X}_5\overline{X}_6X_7 \vee X_1\overline{X}_2X_3\overline{X}_5X_6\overline{X}_7 \vee X_1\overline{X}_2X_3\overline{X}_4X_5X_6\overline{X}_7 \vee X_1\overline{X}_2X_3\overline{X}_4X_6X_7 \vee$ $X_1\overline{X}_2X_3\overline{X}_4X_5\overline{X}_6X_7 \vee X_1\overline{X}_2X_4X_5X_6\overline{X}_7 \vee X_1\overline{X}_2X_4X_6X_7 \vee X_1\overline{X}_2X_4X_5\overline{X}_6X_7 \vee X_1X_2X_4\overline{X}_5\overline{X}_6X_7 \vee X_1\overline{X}_2X_4\overline{X}_5\overline{X}_6X_7$ |
| 4 | $\overline{S}_{PRE}$ | $X_2\overline{X}_6\overline{X}_7 \vee \overline{X}_1X_2\overline{X}_3X_4\overline{X}_5\overline{X}_6X_7 \vee X_1\overline{X}_2\overline{X}_3\overline{X}_5X_6\overline{X}_7 \vee X_1\overline{X}_2\overline{X}_3\overline{X}_4X_5X_6\overline{X}_7 \vee X_1\overline{X}_2\overline{X}_3\overline{X}_4X_6X_7 \vee X_1\overline{X}_2\overline{X}_3\overline{X}_4X_5\overline{X}_6X_7 \vee$ |

| | | |
|---|---|---|
| | | $\overline{X}_1X_2\overline{X}_4\overline{X}_5\overline{X}_6X_7 \vee X_1X_2\overline{X}_4\overline{X}_5\overline{X}_6X_7 \vee X_1\overline{X}_2\overline{X}_4\overline{X}_5\overline{X}_6X_7 \vee \overline{X}_1\overline{X}_2\overline{X}_5X_6\overline{X}_7 \vee \overline{X}_1\overline{X}_2X_5X_6\overline{X}_7 \vee \overline{X}_1\overline{X}_2X_6X_7 \vee \overline{X}_1\overline{X}_2X_5\overline{X}_6X_7 \vee \overline{X}_1\overline{X}_2\overline{X}_5\overline{X}_6X_7 \vee \overline{X}_2\overline{X}_6\overline{X}_7$ |
| 5 | R | $p_2p_6 + p_2p_5q_6p_7 + q_1p_2p_3p_4q_5q_6p_7 + p_1q_2p_3q_5p_6q_7 + p_1q_2p_3q_4p_5p_6q_7 + p_1q_2p_3q_4p_6p_7 + p_1q_2p_3q_4p_5q_6p_7 + p_1q_2p_4p_5p_6q_7 + p_1q_2p_4p_6p_7 + p_1q_2p_4p_5q_6p_7 + p_1p_2p_4q_5q_6p_7 + p_1q_2p_4q_5q_6p_7$ |
| 6 | U | $p_2q_6q_7 + q_1p_2q_3p_4q_5q_6p_7 + p_1q_2q_3q_5p_6q_7 + p_1q_2q_3q_4p_5p_6q_7 + p_1q_2q_3q_4p_6p_7 + p_1q_2q_3q_4p_5q_6p_7 + q_1p_2q_4q_5q_6p_7 + p_1p_2q_4q_5q_6p_7 + p_1q_2q_4q_5q_6p_7 + q_1q_2q_5p_6q_7 + q_1q_2p_5p_6q_7 + q_1q_2p_6p_7 + q_1q_2p_5q_6p_7 + q_1q_2q_5q_6p_7 + q_2q_6q_7$ |

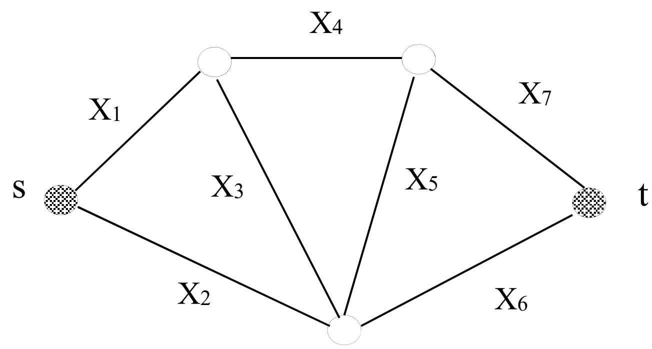


**Fig. 1. A general 5-node 7-element st reliability network.**

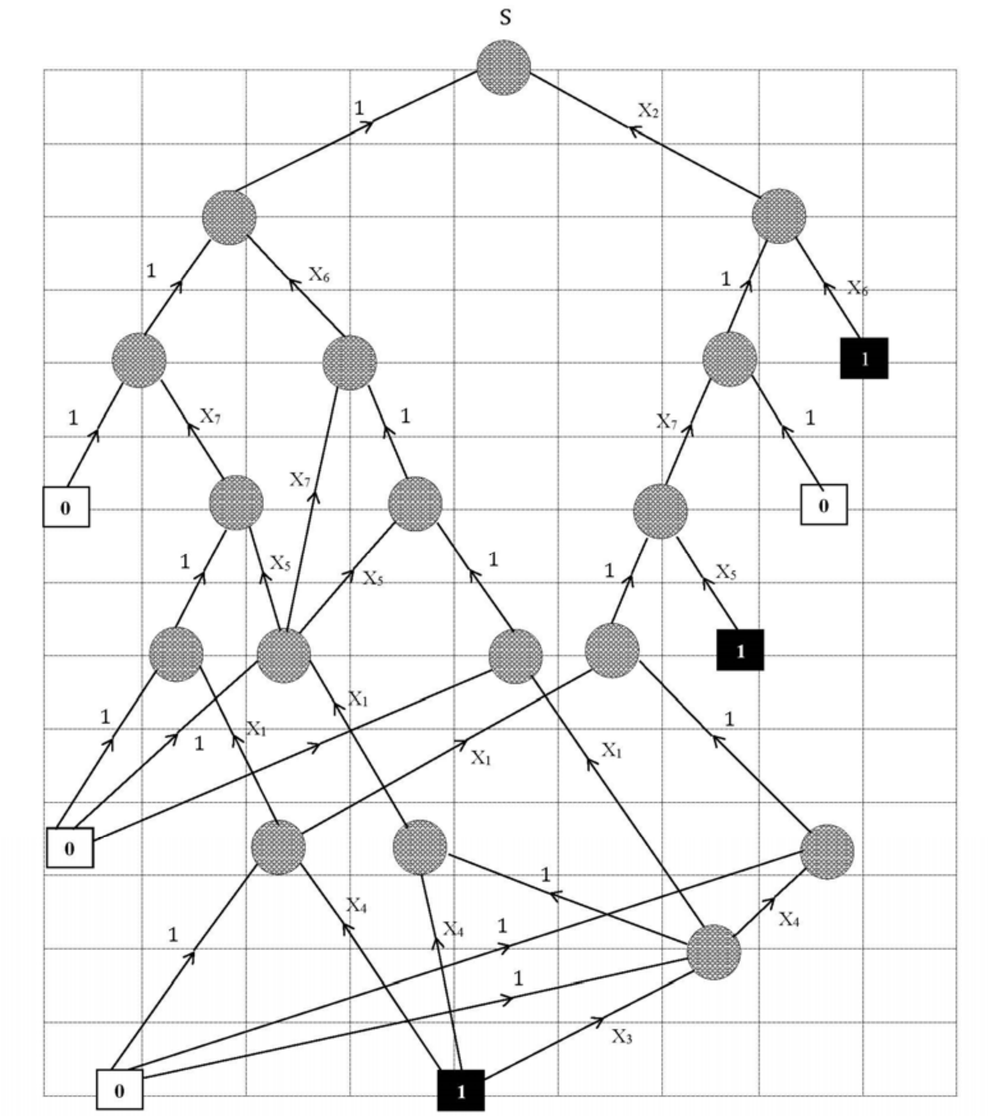


**Fig. 2. A Signal Flow Graph representing system success. Logical addition and multiplication (Oring and ANDing) are implicitly assumed. For convenience, multiple copies of each of the two source nodes of 0 and 1 are used. In addition to the seven minimal paths comprising $S_{minimal}$ in Table 1, the graph produces five non-minimal paths: $X_1X_3X_5X_6$, $X_1X_3X_6X_7$, $X_1X_2X_4X_7$, $X_1X_4X_6X_7$ and $X_1X_4X_5X_7$ .**

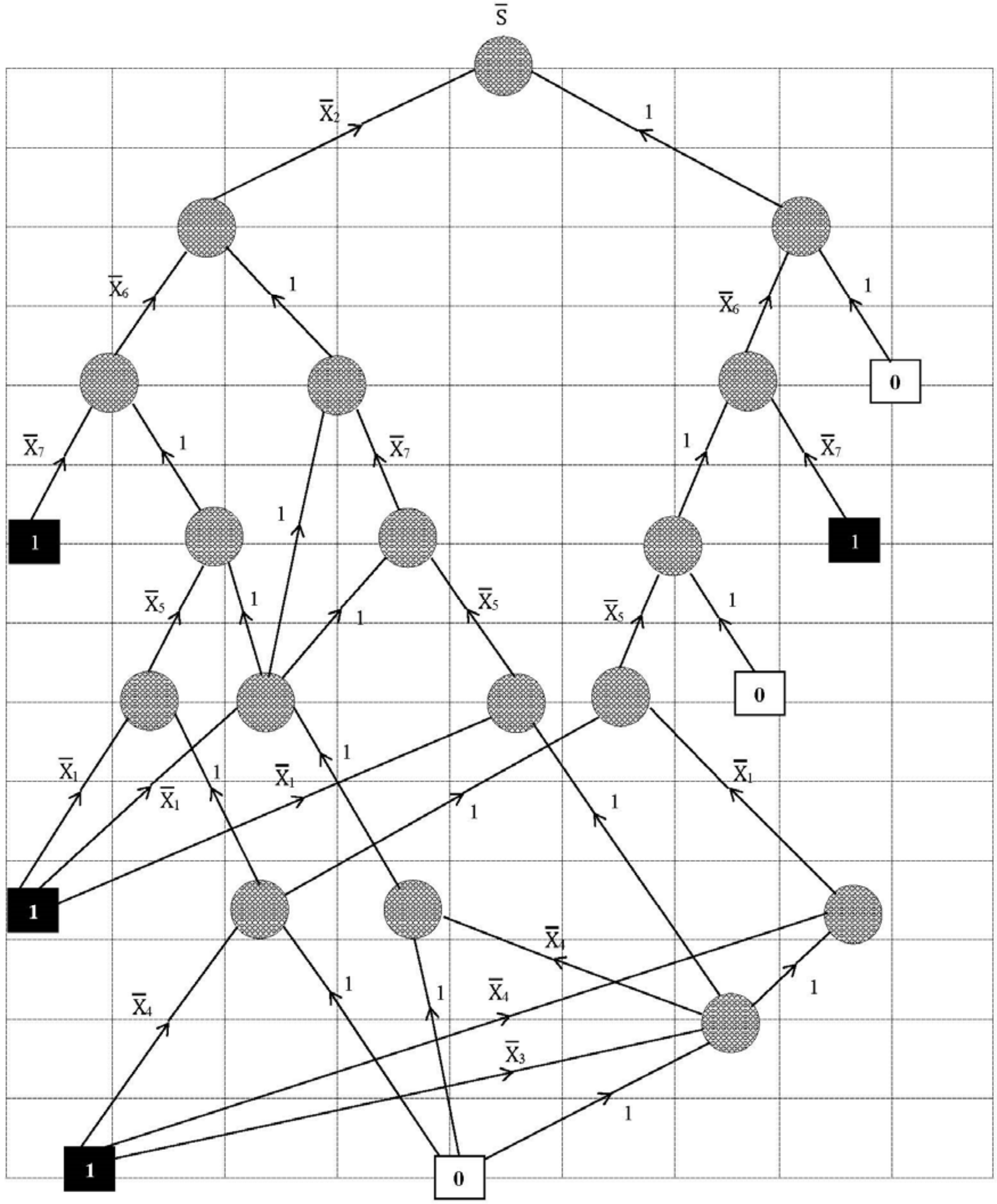


**Fig. 3. A Signal Flow Graph representing system failure. Logical addition and multiplication (Oring and ANDing) are implicitly assumed. In addition to the six minimal cutsets comprising $\bar{S}_{minimal}$ in Table 1, the graph produces nine non-minimal cutsets: $\bar{X}_2\bar{X}_3\bar{X}_4\bar{X}_7$, $\bar{X}_2\bar{X}_3\bar{X}_4\bar{X}_6$, $\bar{X}_1\bar{X}_4\bar{X}_5\bar{X}_6$, $\bar{X}_2\bar{X}_4\bar{X}_5\bar{X}_6$, $\bar{X}_1\bar{X}_2\bar{X}_5\bar{X}_7$, $\bar{X}_1\bar{X}_2\bar{X}_7$, $\bar{X}_1\bar{X}_2\bar{X}_6$, $\bar{X}_1\bar{X}_2\bar{X}_5\bar{X}_6$ and $\bar{X}_2\bar{X}_6\bar{X}_7$.**

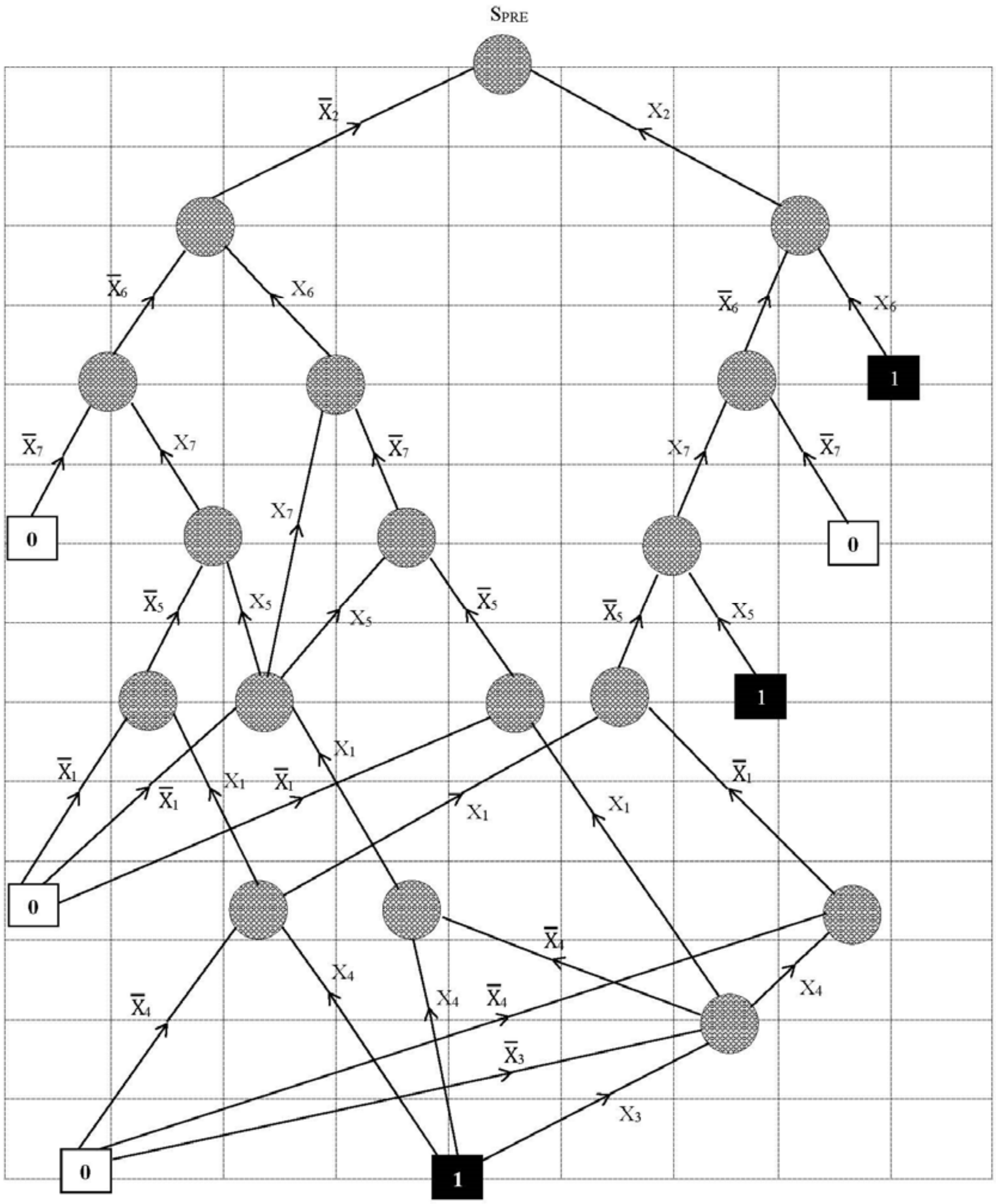


**Fig. 4. A Signal Flow Graph representing a probability-ready expression for system success. Logical addition and multiplication (Oring and ANDing) are implicitly assumed.**

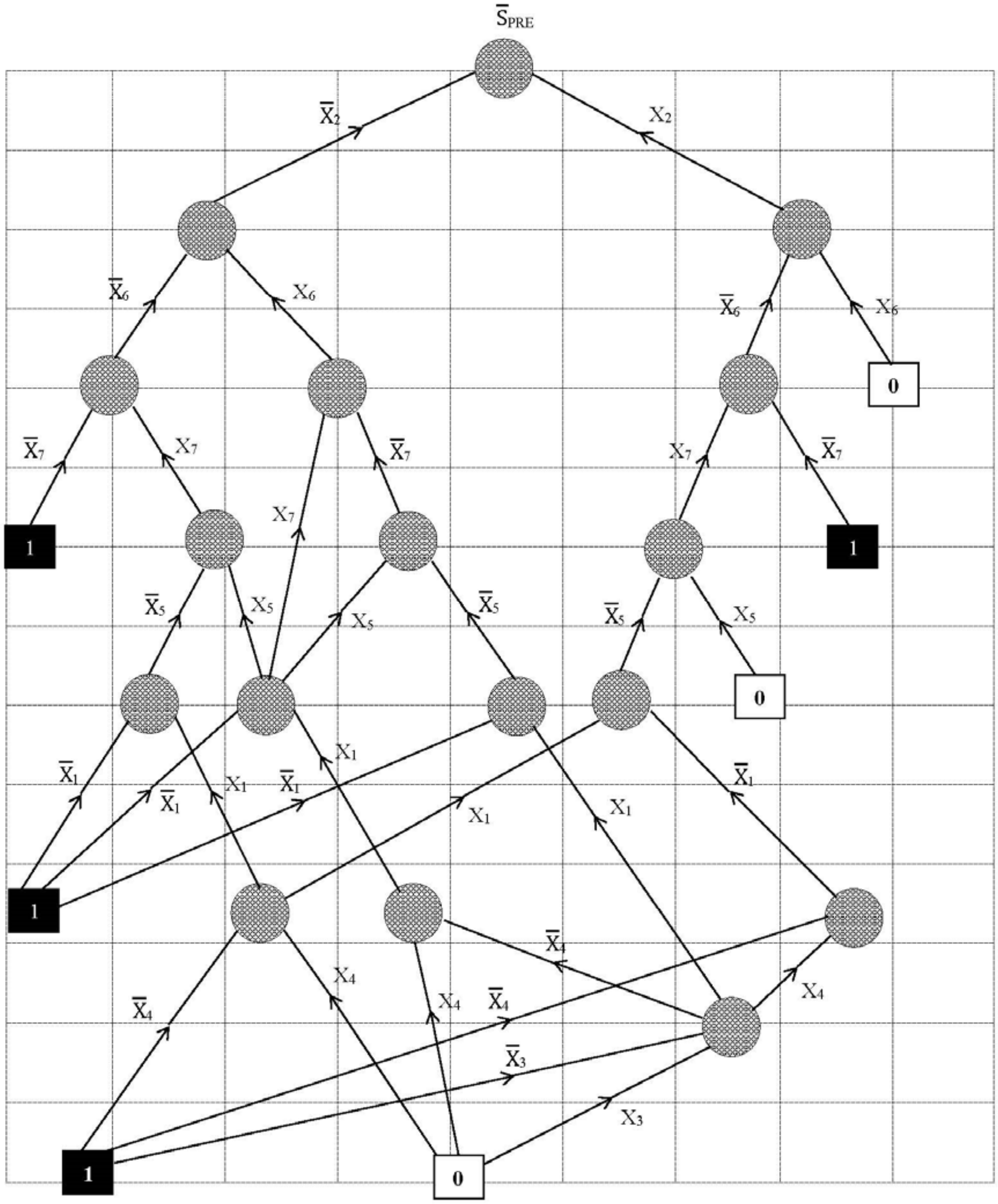


**Fig. 5. A Signal Flow Graph representing a probability-ready expression for system failure. Logical addition and multiplication (Oring and ANDing) are implicitly assumed.**

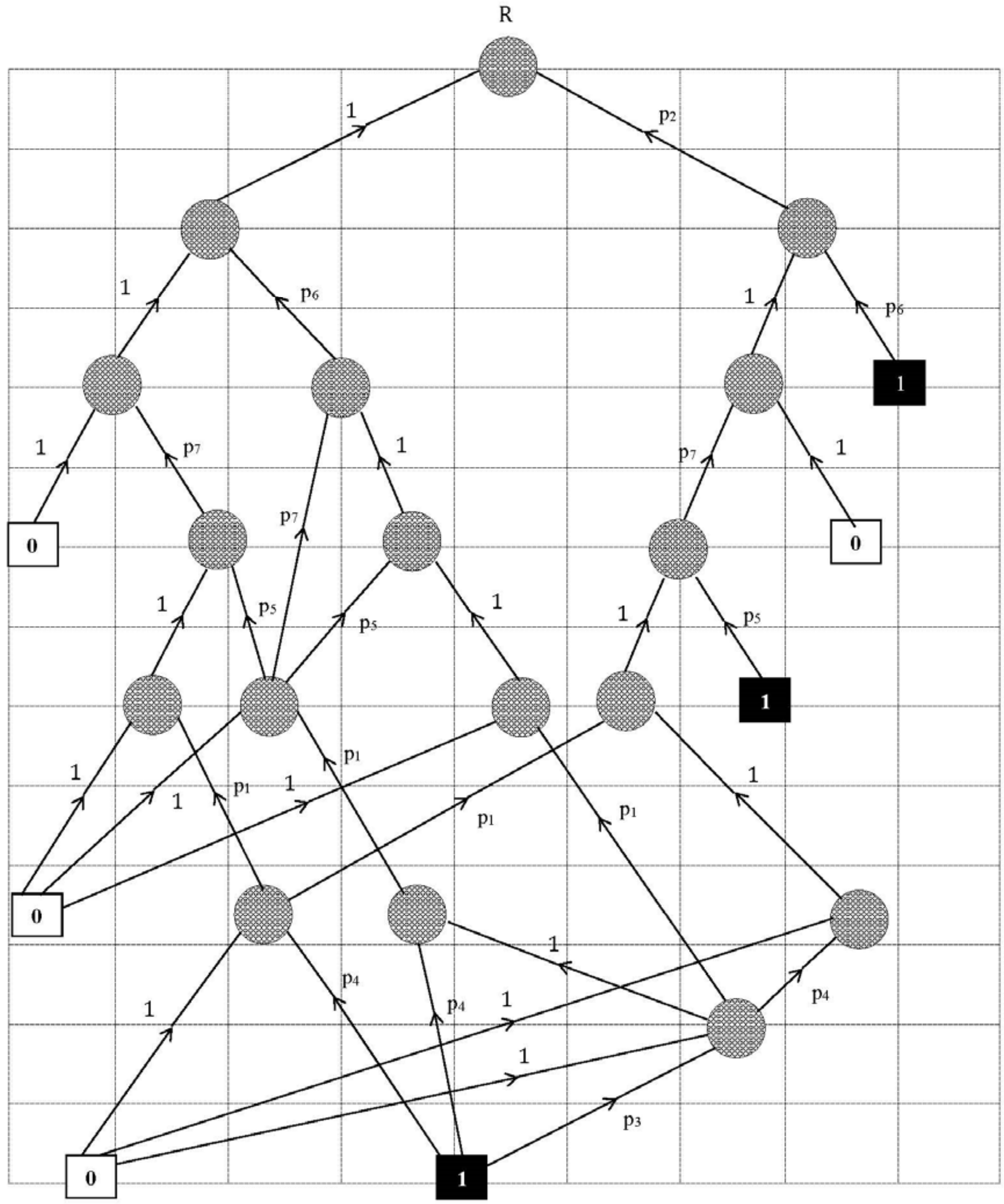


**Fig. 6. A Signal Flow Graph representing system reliability. Usual arithmetic addition and multiplication are implicitly assumed.**

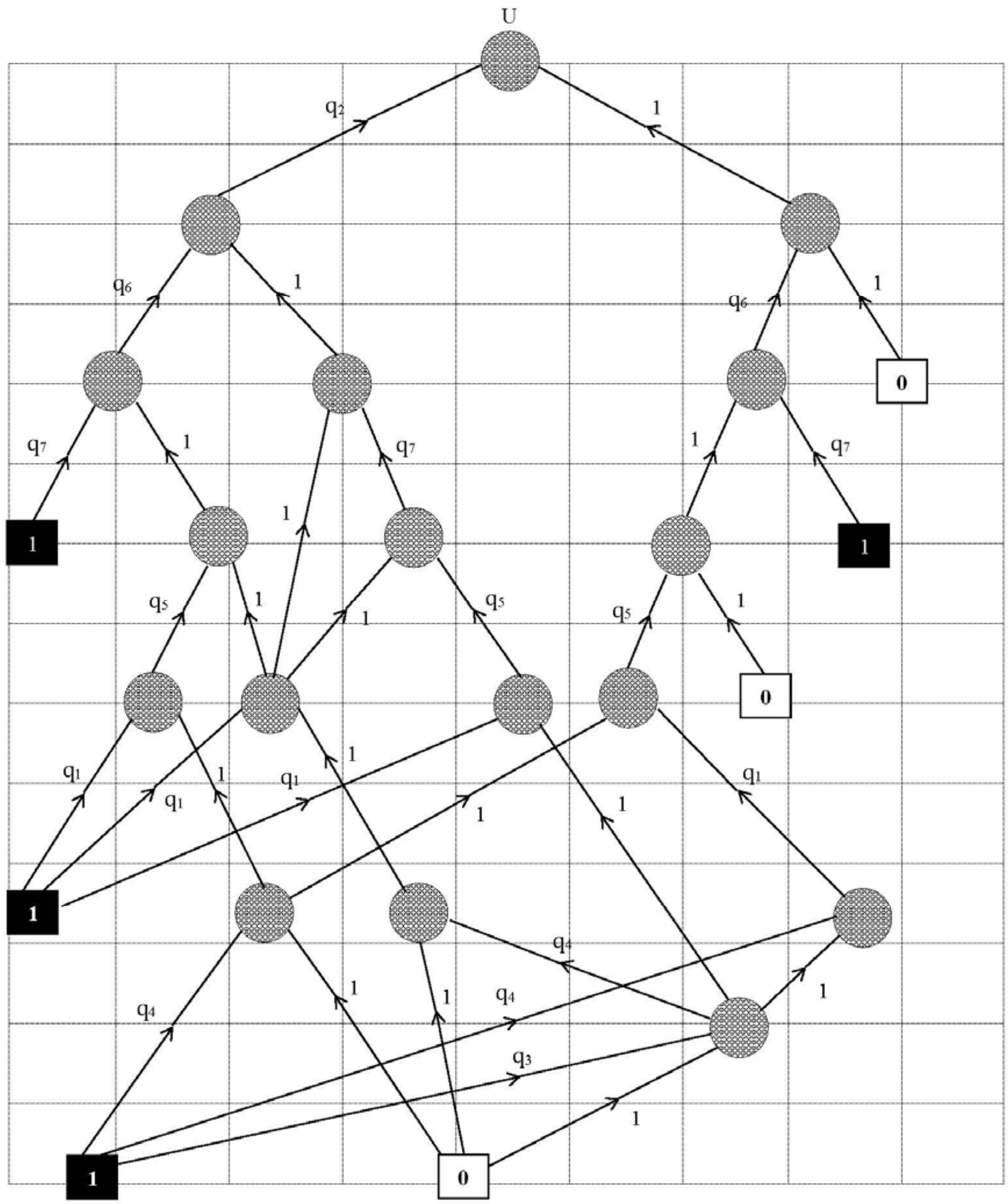


**Fig. 7. A Signal Flow Graph representing system unreliability. Usual arithmetic addition and multiplication are implicitly assumed.**

**References**

[1] **Burroughs, J. L.** and **Happ, W. W.** (1962) *Flow Graph Techniques for Reliability Engineering*. In 1962 IRE National Convention, pp. 338-366

[2] **Misra, K. B.** and **Rao, T. S. M.** (1970) Reliability analysis of redundant networks using flow graphs, *IEEE Transactions on Reliability*, **19**(1): 19-24.

[3] **Aggarwal, K.** and **Rai, S.** (1978) Symbolic reliability evaluation using logical signal relations, *IEEE Transactions on Reliability*, **27**(3): 202-205.

[4] **Kumamoto, H., Henley, E. J.** and **Inoue, K.** (1981) Signal-flow-based graphs for failure-mode analysis of systems with control loops, *IEEE Transactions on Reliability*, **30**(2): 110-116.

[5] **Misra, K. B.** (1992) *Reliability Analysis and Prediction: A Methodology Oriented Treatment*, Vol. 15, Fundamental Studies in Engineering, Amsterdam, The Netherlands: Elsevier Science Publishers.

[6] **Deb, S., Pattipati, K. R., Raghavan, V., Shakeri, M.** and **Shrestha, R.** (1995) Multi-signal flow graphs: a novel approach for system testability analysis and fault diagnosis, *IEEE Aerospace and Electronic Systems Magazine,* **10**(5): 14-25.

[7] **Barker, H. A., Chen, M.** and **Townsend, P.** (1989) Algorithms for transformations between block diagrams and signal flow graphs. In: *Computer Aided Design in Control Systems 1988: Selected Papers from the 4th IFAC Symposium, Beijing, PRC, 23-25 August 1988* (p. 281-286). Elsevier.

[8] **Rushdi, A. M.** and **Alturki A.M.** (2018) Novel representations for a coherent threshold reliability system: A tale of eight signal flow graphs, *Turkish Journal of Electrical Engineering and Computer Sciences,* **26**(1), 257-269.

[9] **Akers, S.** (1978) Binary decision diagrams, *IEEE Transactions on Computers,* C-**27**(6): 509-516.

[10] **Bryant, R.** (1986) Graph-based algorithms for Boolean function manipulation, *IEEE Transactions on Computers,* **35**(8): 677-691.

[11] **Rushdi, A. M. A.** (2010) Partially-redundant systems: Examples, reliability, and life expectancy, *International Magazine on Advances in Computer Science and Telecommunications*, **1** (1): 1-13.

[12] **Rushdi, A. M.** and **Rushdi, M. A.** (2017) Switching-Algebraic Analysis of System Reliability. Chapter 6 in Ram, M. and Davim, P. (Editors), *Advances in Reliability and System Engineering, Management and Industrial Engineering Series*, Springer International Publishing, Cham, Switzerland: 139-161.

[13] **Rushdi, A. M.** and **Alturki, A. M.** (2017) Computation of k-out-of-n System Reliability via Reduced, Ordered Binary Decision Diagrams, *British Journal of Mathematics & Computer Science*, **22**(3): 1-9.

[14] **Rushdi, A. M.** (1986) Utilization of symmetric switching functions in the computation of k-out-of-n system reliability. *Microelectronics and Reliability*, **26**(5): 973-987.

[15] **Rushdi, A. M.** (1991) Comments on: An efficient non-recursive algorithm for computing the reliability of k-out-of-n systems, *IEEE Transactions on Reliability*, **40**(1): 60-61.

[16] **Rushdi, A. M.** (1993) Reliability of k-out-of-n systems, Chapter 5 in K. B. Misra (Editor), *New Trends in System Reliability Evaluation,* Vol. 16, Fundamental Studies in Engineering, Amsterdam, The Netherlands: Elsevier Science Publisher, pp. 185-227.

[17] **Rushdi, A. M.** and **Al-Thubaity, A. O.** (1993) Efficient computation of the sensitivity of k-out-of-n system reliability, *Microelectronics and Reliability*, **33** (15): 1963–1979.

[18] **Rushdi, A. M.** and **Al-Hindi, K. A.** (1993) A table for the lower boundary of the region of useful redundancy for k-out-of-n systems, *Microelectronics and Reliability*, **33**(7): 979-992.

[19] **Rushdi, A. M.** (1990) Threshold systems and their reliability, *Microelectronics and Reliability*, **30**(2): 299-312.

[20] **Rushdi, A. M.** and **Alturki, A. M.** (2015) Reliability of coherent threshold systems, *Journal of Applied Science* **15**(3): 431-443.

[21] **Rushdi, A. M.** and **Bjaili, H.** (2016) An ROBDD algorithm for the reliability of double-threshold systems, *British Journal of Mathematics and Computer Science*, **19**(6): 1-17.

[22] **Rushdi, A. M.** (1987) Efficient computation of k-to-ℓ-out-of-n system reliability, *Reliability Engineering*, **17**(3): 157-163.

[23] **Rushdi, A. M.** and **Dehlawi, F.** (1987) Optimal computation of k-to-ℓ-out-of-n system reliability, *Microelectronics and Reliability*, **27**: 875-896, *Erratum*: *ibid*, **28**: 671 (1988).

[24] **Rushdi, A. M.** and **Goda, A. S.** (1985) Symbolic reliability analysis via Shannon's expansion and statistical independence, *Microelectronics and Reliability*, **25** (6): 1041-1053.

[25] **Rushdi, A. M.** and **AbdulGhani, A. A.** (1993) A comparison between reliability analyses based primarily on disjointness or statistical independence: The case of the generalized INDRA network, *Microelectronics and Reliability*, **33**(7): 965-978.

[26] **Rushdi, A. M. A.** and **Ghaleb, F. A. M.** (2014) The Walsh spectrum and the real transform of a switching function: A review with a Karnaugh-map perspective, *Journal of Qassim University Engineering and Computer Sciences*, **7**(2): 73-112.

[27] **Rushdi, A. M. A.,** and **Hassan, A. K.** (2015) Reliability of migration between habitat patches with heterogeneous ecological corridors. *Ecological Modelling*, **304**, 1-10.

[28] **Rushdi, A. M. A., Hassan, A. K.** and **Moinuddin, M.** (2019) System reliability analysis of small-cell deployment in heterogeneous cellular networks. *Telecommunication Systems*, 1-11.

[29] **Mason, S. J.** (1953) Feedback theory: some properties of signal flow graphs, *Proceedings of the IRE*, **41**(9): 1144-1156.

[30] **Coates, C. L.** (1959) Flow-graph solutions of linear algebraic equations, *IRE Transactions on Circuit Theory*, **6**(2): 170-187.

[31] **Dolazza, E.** (1966) System states analysis and flow graph diagrams in reliability, *IEEE Transactions on Reliability*, R-**15**(3): 85-94.

[32] **Kim, H. K.** (1968) Effects of a vanishing variable on the signal-flow graph, *Electronics Letters*, **4** (23): 501-502.

[33] **Munro, N.** and **McMorran, P. D.** (1970) Signal-flow-graph reduction, Mason's rule and the system matrix, *Electronics Letters*, **6**(23): 752-754.

[34] **Chen, W. K.** (1971) *Applied Graph Theory*, Amsterdam/London.

[35] **Affandi, A. M.** and **Rushdi, A. M.** (1995) An accurate six-port microstrip reflectometer. In the Proceedings of the 1995 SBMO/IEEE MTT-S *International Microwave and Optoelectronics Conference*, Vol. 2, pp. 563-568.

[36] **Teixeira, M. C., Marchesi, H. F.** and **Assunção, E.** (2001) Signal-flow graphs: Direct method of reduction and MATLAB implementation. *IEEE Transactions on Education*, **44**(2), 185-190.

[37] **Golnaraghi F.** and **Kuo B. C.** (2009) *Automatic Control Systems*, Ninth Edition, New York. NY, USA: Wiley.

[38] **Prasad, V. C.** (2010) Compact form of Mason's gain formula for signal flow graphs, *International Journal of Electrical Engineering Education*. **47**: 393-403.

[39] **Prasad, V. C.** (2011) Simplification of signal flow graphs, Circuits, *Systems, and Signal Processing*, **30**(3): 673-682.

[40] **Prasad, V. C.** (2013) On Boolean techniques for non-touching loops of signal flow graphs. *Circuits, Systems, and Signal Processing,* **32**(3): 1443-1453.

[41] **Rushdi, M. A. M., Ba-Rukab, O. M.** and **Rushdi, A. M.** (2016) Multi-dimensional recursion relations and mathematical induction techniques: The case of failure frequency of k-out-of-n systems, *Journal of King Abdulaziz University: Engineering Sciences*, **27**(2): 15-31.

[42] **Rushdi, A. M. A.** and **Alturki, A. M.** (2018) Unification of mathematical concepts and algorithms of k-out-of-n system reliability: A perspective of improved disjoint products, *Journal of Engineering Research*, **6**(4): 1-31.

[43] **Brown, F. M.** (1990) *Boolean Reasoning: The Logic of Boolean Equations*, Kluwer Academic Publishers, Boston, USA.

[44] **Rushdi, A. M.** and **Al-Yahya, H. A.** (2001) Derivation of the complete sum of a switching function with the aid of the variable-entered Karnaugh map. Journal of King Saud University: *Engineering Sciences,* **13**(2): 239-269.

[45] **Rushdi, A. M.** and Al-**Khateeb, D. L.** (1983) A review of methods for system reliability analysis: A Karnaugh-map perspective, *Proceedings of the First Saudi Engineering Conference, Jeddah, Saudi Arabia*, **1**: 57-95.

[46] **Crama, Y.** and **Hammer, P. L.** (2011) Boolean Functions: Theory, Algorithms, and Applications. Cambridge University Press, Cambridge, United Kingdom.

[47] **Ball, M. O.** and **Provan, J. S.** (1988) Disjoint products and efficient computation of reliability, *Operations Research,* **36**(5): 703-715.

[48] **Rauzy, A.** (2008) Binary Decision Diagrams for Reliability Studies, Chapter 25 in Misra, K. B. (Editor), *Handbook of Performability Engineering*, Springer, London, UK, pp. 381-396.

[49] **Mo, Y., Xing, L., Amari, S.** and **Dugan, J.** (2015) Efficient analysis of multi-state k-out-of-n system, *Reliability Engineering and System Safety*, **133**: 95-105.

[50] **Rushdi A. M.** (1985) Map derivation of the minimal sum of a switching function from that of its complement, *Microelectronics and Reliability*, **25**: 1055-1065.

[51] **Shier, D. R.,** and **Whited, D. E.** (1985) Algorithms for generating minimal cut sets by inversion, *IEEE Transactions on Reliability,* R-**34**(4): 314–319.

[52] **Rushdi, A. M. A.** and **Hassan, A. K.** (2021) On the Interplay between Reliability and Ecology, Chapter 35 in Misra, K. B. (Editor), *Handbook of Advanced Performability Engineering*. Springer International Publishing.

# تمثيل نظم المعولية المتسقة بواسطة رسوم سريان الإشارات

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*المستخلص.* يعرف نظام المعولية المتسق (ن ع و) بأنه نظام سببي أحادي الوتيرة ذو اعتماد حقيقي على كل عنصر من عناصره. ومن أهم طرائق دراسة المعولية لمثل هذا النظام توصيف هذه المعولية باستخدام علاقات معاودة يرتبط بها شروط حدية. تقدم ورقة البحث هذه العلاقات المعاودة والشروط الحدية لست كينونات تتعلق بنظام المعولية المتسق. وهذه الكينونات هي (أ) صيغتان تستخدمان أحرفا أحادية الصيغة تعبران عن نجاح النظام وفشله، (ب) صيغتان جاهزتان للتحويل إلى احتمالين تعبران عن نجاح النظام وفشله أيضا، (ج) صيغتان مقتصرتان على الجمع فحسب تعبران عن معولية ولامعولية النظام. يتم تمثيل كل من هذه الكينونات الست بواسطة رسم لسريان الإشارات غير دوراني (عديم الحلقات). يتماثل رسم سريان الإشارات لكل من نجاح النظام وفشله مع مخطط القرار الثنائي المرتب المختزل (خ ق ث ر خ) الذي يمثل هيكل البيانات الأمثل لدالة بولانية. إن العلاقات البينية التي تربط رسوم سريان الإشارات الناتجة توضح إجراءات مثلى لتنفيذ (أ) التحويل الاحتمالي (التحويل الحقيقي) لدالة بولانية، (ب) عكس أو تكملة دالة بولانية، (ج) تحقيق المنافاة أو التعامدية بين الحدود في صيغة مجموع مضروبات لدالة بولانية. تؤول رسوم سريان الإشارة المدروسة هنا إلى مخططات أنيقة للحالات الخاصة للنظم الوافرة جزئيا (نظم ك من بين ن) وللنظم الحدية (نظم ك من بين ن الموزونة). توحي نتائج هذا البحث بضرورة بعث وإحياء استخدام رسوم سريان الإشارة في دراسة معولية النظم لكل النظم العامة المتسقة وغير المتسقة وكذلك للحالات الخاصة منها.

*كلمات مفتاحية*: رسم سريان الإشارات، شرط حدي، علاقة معاودة، نظام المعولية.