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Debt and Equity in a Primary Financial Market: A Theory with Islamic Implications

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ABSTRACT**.** Emphasizing risk averse nature of fund suppliers in an uncertain environment, this paper establishes the possibility of a *riba*-free Pareto optimality in a primary financial market, explaining real life dominance of the mixed debt/equity system in terms of deviation from informational efficiency. Risk premia normally associated with equity returns are definite indicators of risk-aversion on the part of demand side real investors. Under totally undiversifiable risk, a financial investor may prefer equity to risk free asset even though he is more risk-averse than the demand side issuer of equity. It concludes that free market dynamic forces generate a financial order giving privileges to active demanders of funds rather than sympathizing' with risk-averse suppliers of funds as conventional analysis implies. Theoretical claims of efficiency losses in the financial supply process due to Islamization appear to be groundless.

Introduction

This paper addresses a basic financial choice problem involving a risk-free asset[*] and a profit-sharing equity in a single period model, bearing in mind the Islamic injunction against debts with guaranteed returns. The main objective is to show how an Islamic *riba*-free Pareto optimality is possible to obtain in *aprimary* financial market, despite non-conducive conditions *(i.e.* high risk-aversion rates and restricted potential for diversification of investment risk). On this basis we purport to explain why the mixed debt/equity system prevails in actual practice. A similar type of problem has been initially addressed by Masud (1984) through a formulation that did not relate directly to the portfolio theoretic approach.

[*] To sharpen the contrast between debt and equity the farmer is treated as risk-free. The findings, however, are unaffected by introducing risk into debt finance.

In a recent study by Tag El-Din (1991), attention was drawn to an indirect implication of the standard mean-variance portfolio choice theory, which explains the observable mixed debt/equity system in a Pareto optimal set-up. The basic idea depends on the interaction of risk-averse participants in the financial market. Such a risk-aversion thesis' underlies the capital market line (CML) hypothesis which embodies the implication that all financial investors become worse off when the CML (and hence the risk-free asset) is dropped. This implication exploits strictly convex mean-variance indifference curves, but it was shown that unless investment returns are pairwise perfectly correlated, these curves may take other shapes that do not sup port the Pareto optimality; e.g. linearity case.

The present study brings together the direct interaction of supply and demand sides in the primary financial market, where debt and profit-sharing equities are yet to be issued. *Financial investors* constitute the supply side, *while real investors* (entrepreneurs) represent the demand side. We deliberately emphasize risk-averse nature of financial investors in line with the portfolio theoretic property that *borrowers are less risk-averse*[*] *than lenders*. Thus, the basic mean-variance analytical tools of portfolio theory are used, where we specify upward sloping indifference curves for both supply and demand side investors. The former (financial investors) are defined with indifference curves that are steeper than those of the latter (real investors). However, in the standard model for secondary financial markets, the risk-free debt asset is assumed exogenously given' as it is often proxied by the government treasury bills. Our primary market model develops the risk-free asset as an endogenous variable together with the risky profit-sharing equity.

In short, we deal with a free competitive market without government intervention. Demand side real investors (entrepreneurs) and supply side (financial) investors are assumed coming together for the first time.

The Assumed Environment

We start from an abstract environment involving a large number of financial and real investors. Like the Schumpeterian entrepreneurs, real investors have nothing to invest. They completely rely on financial investors for investible funds. In order to focus on the choice problem of financial suppliers, real investors are initially neutralized between issuing a risky profit-sharing equity or a risk-free one. In what follows we describe the main features of our abstract environment.

(i) Informational Efficiency. At the beginning of the period all economic agents are assumed to have homogeneous *expectations* about the uncertain return on investment. This means that the distributional parameters (μ,) are known to all participants. We shall depart from such a condition in conformity with standard portfolio choice theory, though at a latter stage the consequences of relaxing this condition will be examined[*].

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[*] The statement that A is "more risk-averse than B" in the mean-variance sense implies that A has a (uniformly) steeper indifference curve than B.

[*] For a more elaborate definition of informational efficiency in the financial market see Copeland and Weston (1980).

(ii) Distribution of Investment Returns. To make the problem analytically tractable we shall start from a long-term equilibrium with a fixed level of investment risk. This level of risk is assumed measurable and equal to σ, whereas financial suppliers are expected utility maximizers. Hence, at the fixed risk σ funds are supplied to those real investment projects showing the highest possible expected return. Let the ith real investment be characterized by a random return variable X_i (i, 2, ..., n), with parameters:

$$E(X_i) = \mu_i \text{ and } Var(X_i) = \sigma^2$$

then to make portfolio analysis possible through mean-variance, we assume that X_i are normally distributed i.e.

$$X_i - N(\mu, \sigma)$$

The assumption of expected utility maximization entails in the long run that competitive forces would drive out any real investments having an expected return μ_i lower than a market maximum μ.

Given the fixed risk level σ, this will lead to a long run equilibrium where all $\mu_i < \mu$ have already been driven out of the market. Thus, assuming independent returns, our point of departure will be characterized by a set of random returns (X_1, X_2, X_n) that are i.i.d. normal variables, i.e.

$$X_i - N(\mu, \sigma)$$

(iii) Equity Share and Risk-Free Rate. Also, it is impossible under the long-term (informationally-efficient equilibrium) to have more than a single risk-free rate, r. A fixed profit-sharing ratio, a, will be assumed, and the return variable $\{ X_i \}$ are taken as unit dollar net returns. Thus, any financial supplier of a unit dollar is offered a random share Y_i,

$$Y_i = aX_i (i = 1, ..., n),$$

from the net dollar return of the ith. This yields the following risk/return parameters of the issued equity

$$m = E(Y_i) a\mu, \text{ and}$$
$$S = \sqrt{Var}(Y_i) a\sigma$$
$$0 < a < 1$$

Accordingly the profit-sharing equity will be denoted by the two components vector (m, s), whereas the risk-free debt asset will be denoted by (r, o).

(iv) Consequences of the i.i.d. Property. It is notable that the i.i.d. property of investment returns greatly simplifies the shape of the efficient portfolio, given any fixed ratio, a. Since m is fixed, the process of risk-diversification merely reduces the risk component, s, of the equity. Hence, for any fixed profit-sharing ratio, a, the risk/return parameters of share equity (m, s) shall be translated into the efficient frontier through risk diversification in the Markowitz sense [Markowitz (1952)], as:

$$(m, s) \qquad\qquad (m_e, s_e)$$

where $m_e = m$ but $s_e < s$ as a result of diversification.

Reduction of risk due to diversification is allowed only up to a minimum value $s_e > 0$, which denotes *undiversifiable* risk.

(v) Relevance of Moral Hazards. The problem of moral hazard is assumed away in this paper due to reasons outlined by Tag El-Din (1990). Our intent is to restrict attention to the "risk-averse thesis" of portfolio theory. The "moral hazard thesis" is however irrelevant as it cannot explain the choice problem involving bonds and equities.

Plan of the Study

Our findings are based on a simple geometrical device which we have developed to bring forth the direct interaction of supply and demand sides in the primary financial market. The new concept of Investors' Share Box (ISB) is introduced in part (1) of the paper. ISB is constructed by utilizing basic analytical tools of mean-variance portfolio choice theory, with some adaptations. Perhaps the most significant modification is the treatment of the (risk-free) interest as an *endogenous* variable*. Also real investors are shown to have identical indifference curves as a result of their initial neutrality under conditions of the assumed environment. We rely on geometric intuition in the exposition of the theory, rather than sophisticated calculus. This is deliberately adopted since new ideas are best revealed in simple intuitive terms.

Part (2) focusses on the supply side participants of the market, keeping real investors effectively neutralized. In part (3), we shift to the demand side, relaxing the condition of informational efficiency and, hence, the neutrality of real investors. Finally, we briefly pinpoint the main concluding remarks.

Part - 1
Portfolio Analysis within ISB

Basic adaptations of the standard mean-variance portfolio analysis are needed to suit the special features of our financial choice problem. In fact, subsequent analysis is demonstrated within the Investors' Shares Box (ISB), which describes the primary financial market in terms of direct interaction between the two types of investors (*real and financial*). The ISB is defined over a closed risk/return space, for two types of financial assets: equity share and risk-free debt. It combines investors' preferences with the objective investment opportunities, in a manner that brings to a sharp focus the choice problem between the risk-free asset and the risky profit share.

1.1 The ISB and The Basic Shares Possibility Set

The ISB, essentially, represents the different possibilities of sharing a total expected net return of a unit dollar investment, characterized by distributional parameters μ and

* The risk-free rate in the standard portfolio theory is assumed given exogenously. In our present study it is taken to depend on the real investors' attitude towards risk.

σ, as between a real investor and any financial investor. Figure 1 is self-explanatory. Note that the south-western corner of the box represents the point of origin for financial investors' risk/return axis, while the north-eastern corner stands for a typical real investors' point of origin.

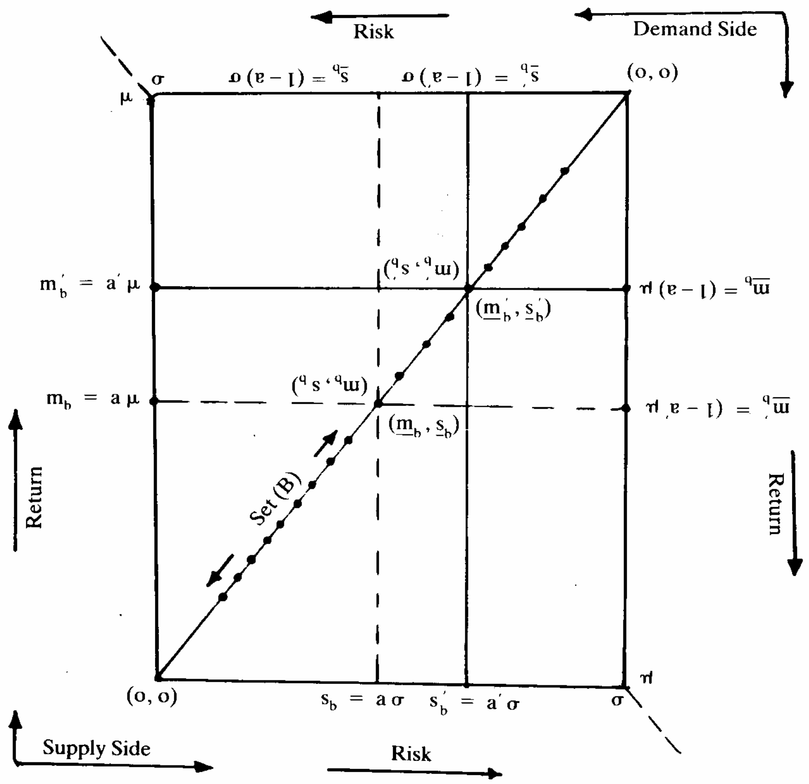

FIG. 1. The Investors' Shares Box and the Basic Shares' Set (B)

NB: o < a < a < 1

That is, if a financial investor is offered a return share (a μ) the real investor is left with the remaining share (1 - a) μ. As it is shown, the return shares of the two partners are determined by the horizontal line connecting the opposite return axes.

Similarly, the risk-shares of either partner are determined by the corresponding vertical line connecting the opposite risk axis. The two perpendicular connectors intersect at a specific point which is a single element in the basic shares possibility set (B), defined for either partner as:

$$B = \{(m_b, s_b,) / m_b = a\mu, s_b = a\sigma, 0 < a < 1\}$$

Note that the set (B) is represented by the main diagonal of the ISB

1.2 The Efficient Shares Possibility Set (E)

The set (E) is only relevant for financial investors who gainfully exploit the opportunity of diversifying investible funds over the full real investors[*]. Thus, any given basic payoff:

$$(m_b, s_b) \in B$$

can be translated through the diversification process, into the corresponding efficient payoff:

$$(m_e, s_e) \in E$$

noting that $m_e = m_b$), due to the i.i.d. property, of returns, while s_b is reducible to the (minimum) undiversifiable level $s_e = a\sigma_e$ for any fixed ratio, a. The efficient shares possibility set is, thus, defined as:

$$E = \{(m_e, s_e,) / m_e = a\mu, s_e = a\sigma_e, 0 < a < 1\}$$

and it is represented in Figure 2 by the steeper line to the left of the main diagonal of the ISB. Note that the absolute gain from diversification, g_a, rises linearly with the sharing ratio, a, since

$$g_a = s_b - s_e = a(\sigma - \sigma_e) = ag_1$$

Clearly the line representing set (E) becomes steeper for larger values of g see Figure 2.

1.3 The Universal Possibility Set (T)

The set (T) represents all possible (m, s) equity shares obtainable through profit-sharing with or without diversification. It includes not only sets (B) and (E), but also the intermediate diversified shares where funds are only allocated over a limited range of real investments. Thus, there is yet a third set of possible non-basic shares, with the risk component s_b being reduced to some s_n by partial diversification, resulting in:

$$\sigma > s_b > s_n > s_e > 0$$

Thus the non-basic shares possibility set (NB) is defined as:

$$NB = \{(m_b, s_n,) / m_n = a\mu; s_e < s_b; 0 < a < 1\}$$

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[*] See Appendix (B).

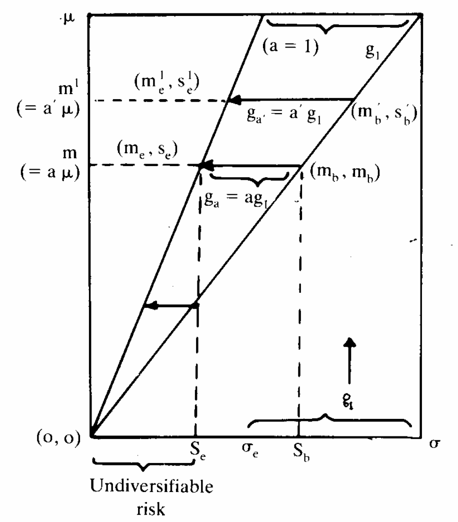
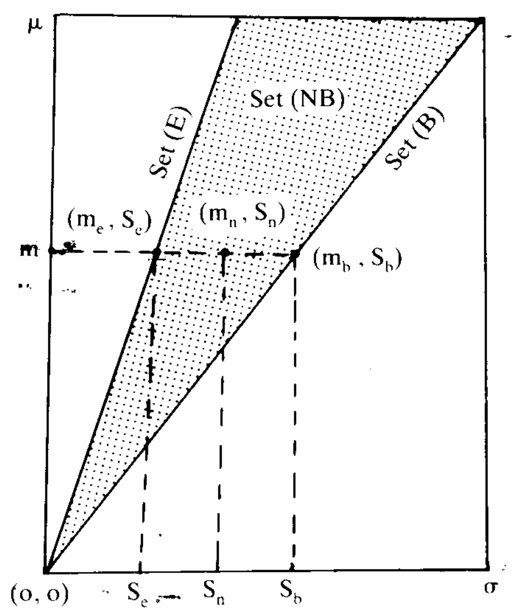


FIG. 2. Translation of Basic Shares into Efficient Set (E).
NB: $m_b = m_e (= m)$, but $S_e < S_b$
$g_1 = \sigma - \sigma_e$; $o < a < a' < 1$

and it is represented by the shaded region between the diagonal of the ISB [i.e. the set (B)]; see Figure 3.



T = B + NB + E
FIG. 3.

Accordingly, the universal shares possibility set is given by the union:

$$T = (B) \cup (NB) \cup (E)$$
$$= B + NB + E$$

since the sets (B, NB, E) are mutually disjoint. It is immediately notable that the set (T) compares to the investment opportunity set of the standard mean-variance portfolio choice theory, whereas the set (E) stands for its efficient frontier.

1.4 The Preferences Structures within The ISB

In Figure 4 it is shown how a risk-averse financial investor maximizes expected utility, by taking risk diversification to its fullest extent. That is, by choosing from the efficient frontier (E) of set (T). This result is true for any assumed curvature of the up wards sloping indifference curve and not necessarily the convex shape. We shall, however, introduce an essential rationality axiom in terms of the slopes of m-s in difference curves.

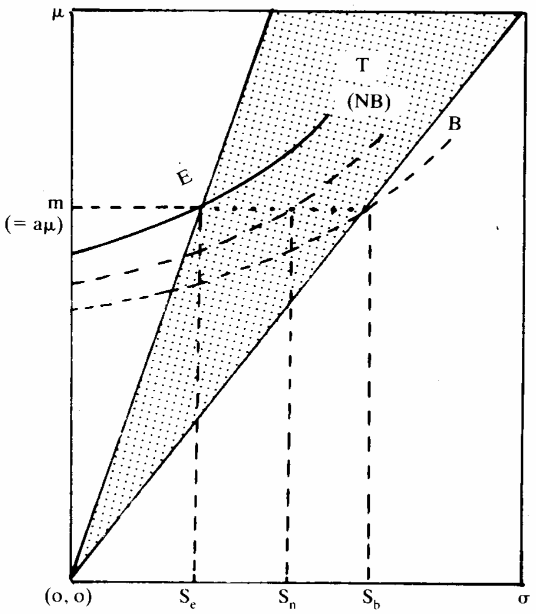


FIG. 4. Maximization of expected utility, by taking risk diversification (at a given sharing ratio) to its fullest extent up to the efficient frontier (E).

1.5 A Rationality Axiom

All that we need to impose on the preferences' structures for investors, is the following axiom:

Any share in profits is better than no share, and a larger share in profit is strictly preferred to a smaller one.

Thus, given the sharing ratios 0 < a < a' < 1, this implies the strict preference relations:

$$m'_b, s'_b \rightarrow (m_b, s_b) \rightarrow (0,0)$$

where $m'_b = a'\mu, s'_b = a'\sigma_e, m_b = a\sigma,$ and $s_b = a\sigma_e$

and the two (m, s) vectors are elements in the basic shares at (B). The class of potential indifference curve is, hence, restricted by ruling out the ones violating the above rationality axiom. Figure 5(a), (b), demonstrates cases which violate this rationality axiom. The first case (a) describes a family of m-s indifference curves where a share in profit is negative utility generating. That is, where nothing is preferred to any share in profits, and that a smaller share is preferred to a bigger share. The second case (b) describes one where the bigger basic share (m'_b, s_b) falls on the same indifference curve as the smaller share (m_b, s_b).

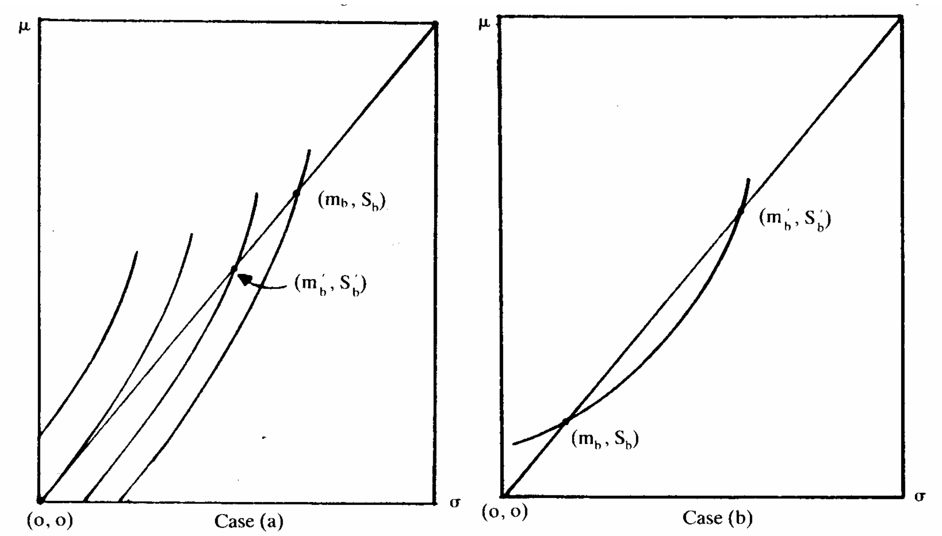


FIG. 5. Cases violating the rationality axiom: $(m'_b, S'_b) \rightsquigarrow (m_b, S_b) \rightsquigarrow (o, o)$.

To rule out such cases, we shall introduce the following rationality restriction uniformly within the ISB

$$0 < \left(\frac{dm}{ds}\right)_{u=uo} < \beta = \frac{\mu}{\sigma}, (\text{for } 0 < s < \sigma; 0 < m < \mu)$$

where $(dm/ds)_{u=uo}$ is the slope of the (m-s) indifference curve at any fixed expected utility level U0 = U (m, s) and β is the Slop* of the main diagonal of the ISB. Hence, a rational risk-averter must have a family of upwards sloping (m-s) indifference curves

* The slope B is related to the coefficient of variation (C.V.) by the simple relation β = 1/C.V. The value of β is indeed a `social' parameter that can be assumed endogenously given as a reflection of entrepreneurs attitude towards risk. As a result riskier investments with relatively largere' cannot be chosen.

with slopes smaller than β. Of course this condition must also apply to the real investor. It will shortly be shown that unless the above rationality restriction is satisfied, the real investor cannot offer positive interest on debt.

We now turn to exploit the above characterization of the ISB, in the choice problem between the risk free security and the risky share equity.

Part -2
Supply Side Perspective

In order to focus on the choice problem by financial investors, it is appropriate to neutralize the real investor's (entrepreneurs') preference between issuing debt security or share equity. This implies, in the Miller-Modigliani sense, that the cost structure of capital is irrelevant to the real investor. Using the ISB, such neutralization is achieved where the cost of debt [i.e. the interest payment (r, o)] and the cost of equity (i.e. the profit share (m_b, s_b) fall on the same indifference curve of the real investor; see Figure 6. However, to embody the necessary ex-ante property

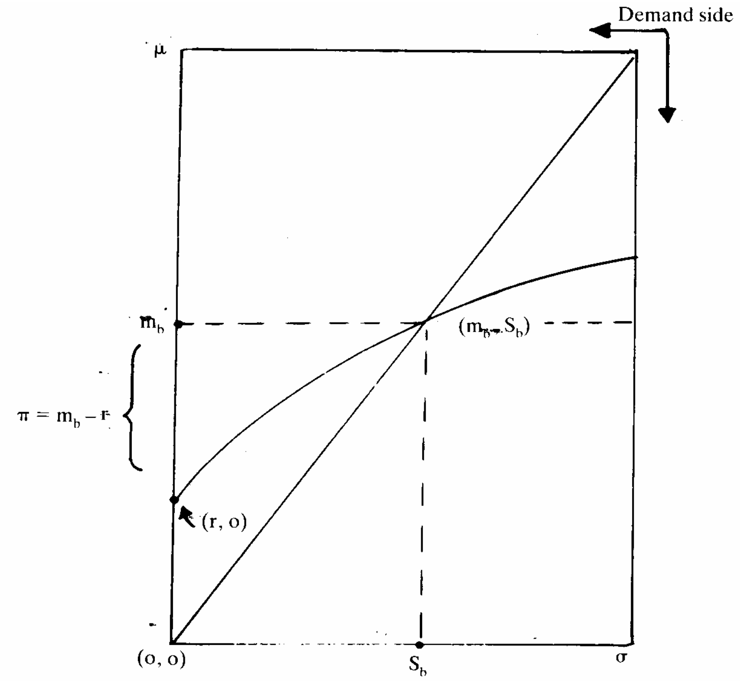


FIG. 6. Real investor indifferent between issuing equity share (m_b, s_b) or risk-free asset (r, o). The steeper is the indifference curve the lower will be the interest rate, r. To maintain r > o, it is necessary to maintain the 'rationality axiom.'.

$$m_b - r > 0$$

the real investor must also have an upwards sloping indifference curve. That is, he must be risk-averse. For, if real investors were risk-neutral, with horizontal (m-s) indifference curves, this will result in a zero risk premium:

$$\pi = m_b - r = 0$$

But in this case, unless all risk is diversifiable (i.e. s_e=0), no one will choose the risky equity; see Figure 7. Here all financial investors with upwards sloping indifference curves, are better off with the risk free asset implying that equities will be completely driven out of the financial market.

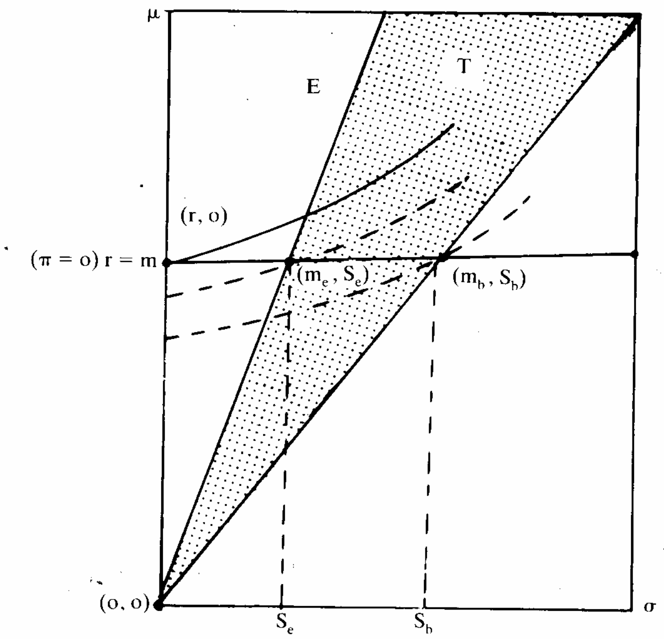


FIG. 7. A risk-neutral real investor, Π = o.

NB: Any risk-averse financial investor would prefer the risk-free asset (r, o) to the risky equity (m_b, s_b), so long as undiversifiable risk exists; i.e. $s_e > o$.

We are, thus, led to the first result below:

Result (1): Under the assumed informationally-efficient environment the existence of positive *ex-ante* risk-premia is a definite indicator of risk-aversion on the part of real investors (entrepreneurs).

We shall therefore stick to the assumption that real investors possess upwards sloping (m-s) indifference curves, like financial investors. It is also necessary to impose our *rationality axiom*, on the slope of any real investor's indifference curve, which now becomes a necessary (though not a sufficient) condition for the existence of a positive risk-free rate, r^*. Moreover, the existence of a single risk-free asset and a single equity

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* It is easy to show how the rationality axiom' is necessary but not sufficient for the existence of r> 0, as just described. In fact for very low values of, a, real investors cannot offer any r > 0, even where the above axiom is maintained.

in the market, can only be justified by the assumption that *all real investors possess identical (m-s) indifference curves*, and are, hence, equally indifferent between the issuance of equity or debt security. In this manner, attention will be focused on the supply side. And to emphasize the problem of negative attitudes towards risk, we shall allow for greater risk-aversion on the supply side compared to the demand side. This implies that all (m-s) indifference curves of financial investors are at least as steep as that of the real investor.

2.1 Equity Shares without Diversification

At the outset, it is appropriate to inquire into the unrealistic limiting theoretical case where all risk is non-diversifiable (s_e=0). It is interesting to see whether diversifiability of risk is indeed a necessary condition for financial markets to accommodate risky equities.

This situation is depicted in Figure 8 where the sets (E) and (NB) are empty (i.e. E = NB = ϕ) and the universal shares possibility set (T), consists only of the basic shares set (B). Let us first consider Figure 8(a). In this figure the real investor and all financial investors are assumed to possess convex (m-s) indifference curves, thus implying increasing risk-aversion. For simplicity, a single financial investor is shown with steeper indifference curves at all levels compared to a real investor.

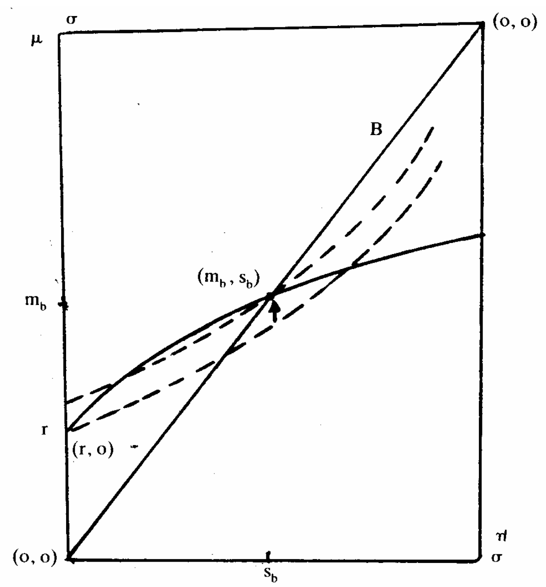

FIG. 8(a). Financial investor with uniformly steeper indifference curve than real investor. Nonetheless, he prefers (m_b, s_b) to (r, o).

As the figure shows, this financial investor still prefers the basic equity share (m_b, s_b) to the risk-free asset (r, o). Interestingly this reveals that even though risk is undiversifiable, the *more* risk-averse individuals are likely to prefer the equity share to the risk-free asset. This follows from the special case of convex curves.

Alternatively, if linear (m-s) indifference curves are adopted to denote constant risk-aversion, Figure 8(b) provides a different implication. In this case the linear extension of the real investor's indifference curve provides an indifferent curve of a possible financial investor. This ensures that the latter has the same (constant) rate of risk-aversion as the real investor. However, it is obvious that this individual is indifferent between the equity share (m_b, s_b) and the debt asset (r, o), as they both fall on one indifference curve. It follows therefore that any other individual with a some what higher rate of risk-aversion than real investor [see Figure 81, would strictly prefer the risk-free asset to the equity share.

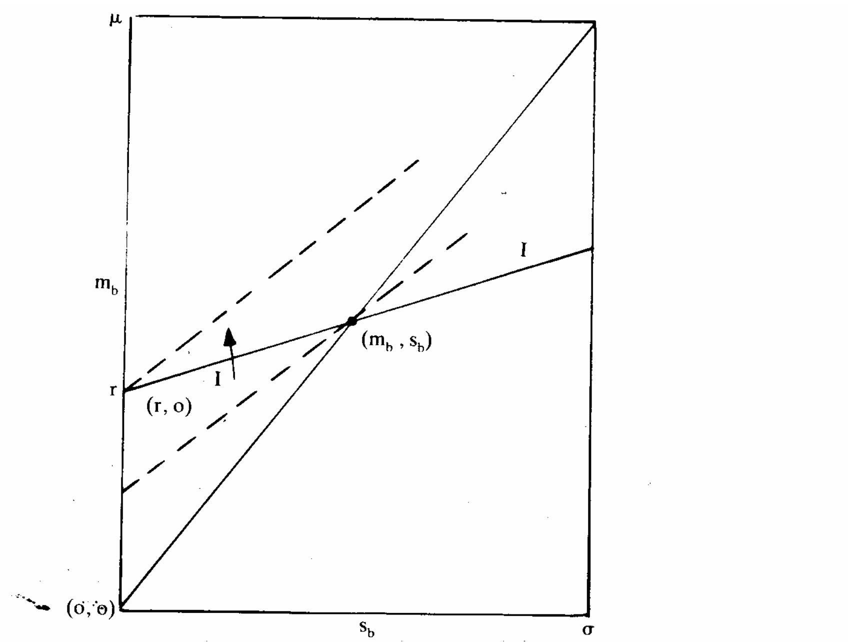

FIG. 8(b). Cases of constant risk-aversion. Any financial investor with a steeper indifference curve than real investor, would prefer (r, o) to (m_b, s_b).

This demonstration justifies the second result:

Result (2): Diversifiability of risk is not a necessary condition for the choice of risky equity share under the assumed environment even though financial investors are more risk-averse than (neutralized) real investors. That is:

a) If increasing risk-aversion prevails (i.e. convexity case) investor might, nonetheless, prefer risky equity to risk-free debt.

b) If constant rates of risk-aversion prevail (i.e. linearity case), then all financial investors would prefer debt to equity.

It may look rather surprising that the stronger case for the preference of equity without diversification is supported by the commonly adopted property of increasing risk-aversion. We should, however, recall that we are dealing with a *primary* financial market, where the risk-free asset is *recognized* in terms of demand side real investors attitudes towards risk.

2.2 Allowance for Diversifiability

We now consider the more realistic case where risk is partly diversifiable with a persistent non-diversifiable residual $s_b > 0$. In the remainder of this section we shall assume that all (m-s) indifference curves are linear as in Figure 8(b), mainly a means to simplify the analysis Stronger implications can be derived with convex in difference curves.

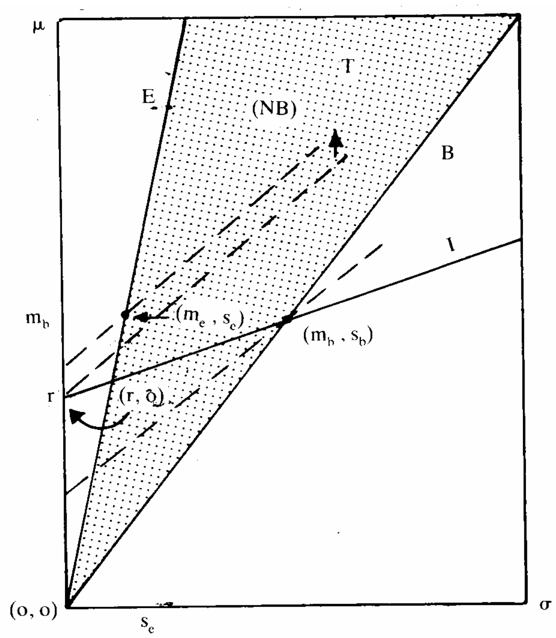

FIG. 9. As diversification is introduced the order of preference is reversed for the individual in Figure 8(b). He now prefers equity to debt.

Diversifiability of risk is now represented in Figure 9 by the set (T) which is extended to include the efficient frontier (E) and the intermediate non-basic set (NB). Note that the individual in Figure 8(b) who preferred debt to equity, has now reversed his preference. This is a typical case of individuals with higher rates of risk-aversion (relative to the real investor), who find diversifiability of risk sufficient to make the issued equity share more attractive than the risk-free asset. We are, hence, led to the following familiar result:

***Result** (3):* Although diversifiability of risk may not be necessary, it is sufficient to induce some financial investors, with linear indifference curves, who would other wise prefer debt to equity, to reverse their order of preferences.

Now, the crucial question, which is pertinent to Islamic economics, is whether a financial market based exclusively on profit-sharing, may be derived despite the wide range of high risk-aversion rates and the persistence of undiversifiable risk! We shall thus turn to explore stronger results.

2.3 Potentially Superior Assets and the Driving Out of Risk-Free Assets

Let us first introduce the idea of a relatively superior asset and then define its potential set (R).

Definition: A financial investment asset (m, s) is defined as superior relative to a rival risk-free asset (r, o), if the former drives the latter completely out of the informationally efficient financial market.

Of course this requires that every *rational* financial investor, no matter how risk averse, will strictly prefer that investment asset to the specified risk-free debt asset. As a special simple example of this concept, in Figure 10(a), note that every (m, s) asset contained within the upper triangle of the ISB is superior relative to the rival zero interest debt asset (o, o). In this simple case the set (R) of relatively superior as sets consists of two main types of potential assets:

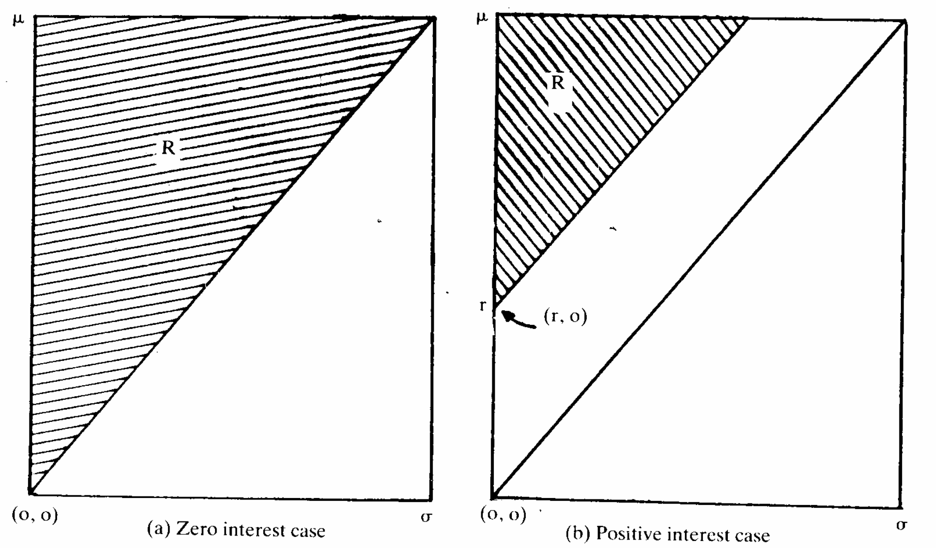

FIG. 10(a, b). Sets of relatively superior assets.

i) The first type is the positive interest risk-free assets {(r, o)}. The inclusion of this type of assets in the set (R) is trivially obvious. They represent the upper boundary of set (R), which is the vertical r-axis of the ISB.

ii) The second type is the potential (m, s) risky assets with risk components{s} falling in the range $o < s < s_b$ for any fixed m-recalling that the maximum S_b is the risk component of the basic equity share $(m, s_b) \in B$, as defined on the main diagonal of the ISB.

The inclusion of set (B) in the set (R) is justified by the previously stated rationality axiom: that any (basic) share in profits is better than nothing. And since any (m, s) point to the left of the main diagonal is strictly preferred to the corresponding (m, s_b) point on the diagonal, it follows that the latter represents the lower boundary of the set (R).

Now if the implied rationality restriction is always maintained (i.e. indifference curves always have smaller slopes than B) then it is always possible to define a set (R) of superior assets (m, s) relative to rival debt assets for any rate r in the range $0 < r < \mu$. Naturally, as r rises, the corresponding set (R) diminishes. In Figure 10(b) we exemplify for a region (R) relative to risk-free asset (r, o). In this Figure the lower boundary of set (R) is contained within the upper triangle of the ISB, parallel to its main diagonal. To verify that the given set (R) is indeed one of relatively superior as sets in the way they are just defined, we only need to establish this property for the lower boundary. Thus, we may prove the following lemma:

***Lemma*:** Given any line within the upper triangle of the ISB, and parallel to its main diagonal, the rationality axiom implied that every potential (m, s) risky asset defined on that line is 'superior' relative to the risk-free asset (r, o) defined on the same line.

[Proof is given in Appendix (A)]
This lemma provides the basis for the following result:

Result (4): No matter how risk-averse are rational financial investors, there are always potentials of risky assets that may drive the rival risk-free assets completely out of the market.

2.4 Relatively Superior Equity Shares

It remains to substantiate Result (3) by showing how potentially superior assets can be realized in terms of the equity shares contained in set (T), recalling that T = B + NB + E. It is immediately notable that no basic equity share, $(m_b, s_b) \in B$, can also be a member of set (R) where r > 0. That is,

$$R \cap B = \phi$$

unless r = 0. Only in the latter case where T R, it follows automatically that $R \cap B = B$.

Thus, to include equity shares in set (R) for any r > 0, the basic equity issue (m_b, s_b) must be translatable through risk diversification into an element $(m_b, s_e) \ni E$ (or $m_n, s_b)$ $\in NB$ such that the translated element becomes a member in set (R).

In Figure 11(a), (b), we provide two possible situations as follows:

(a) The null case $(R \cap T = \phi)$ relative to a fixed interest rate r $= r_1 > 0$. At the given extent of diversifiability (g), there is no profit-sharing ratio which can yield a superior equity share (m, s)$\in$ T, relative to the risk-free asset (r_1, o) - not given the maximum ratio a = 1, yielding $m = \mu$

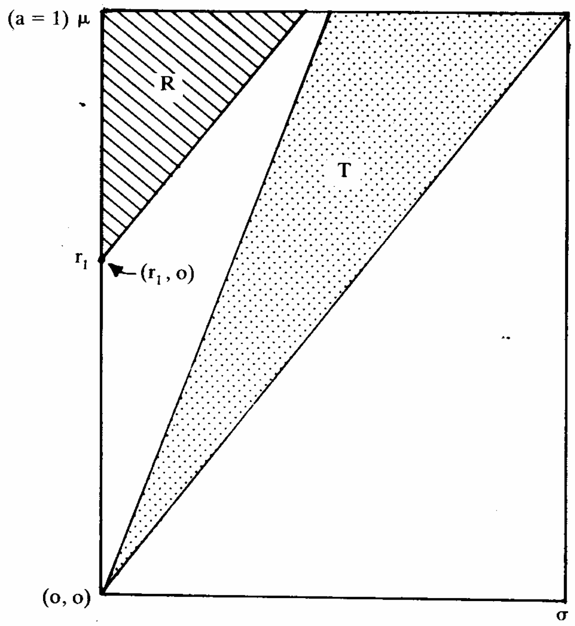

FIG. 11(a). No profit-sharing ratio may yield a 'superior equity share' not even the full share a = 1.
$R \cap T = \emptyset$ (null case)

(b) *The non-null* case: $(R \cap T = Q)$ relative to a lower interest rate $r_2 < r_1$, but with the same extent of diversifiability as given in (a) above.

The necessary condition for the existence of a non-null set Q relative to any fixed interest rate is the intersection of the efficient frontier (E) with the lower boundary of set (R). This point of intersection, when it exists, determines the smallest profit-sharing ratio, a, (on financial investors' axis) that is necessary to yield a relatively superior equity share.

2.5 Exclusive Dominance of Profit Sharing

Now that a non-null set (Q) may exist relative to any risk-free asset (r, o), it remains to show how in the assumed equilibrium, profit-sharing may drive out risk-free assets.

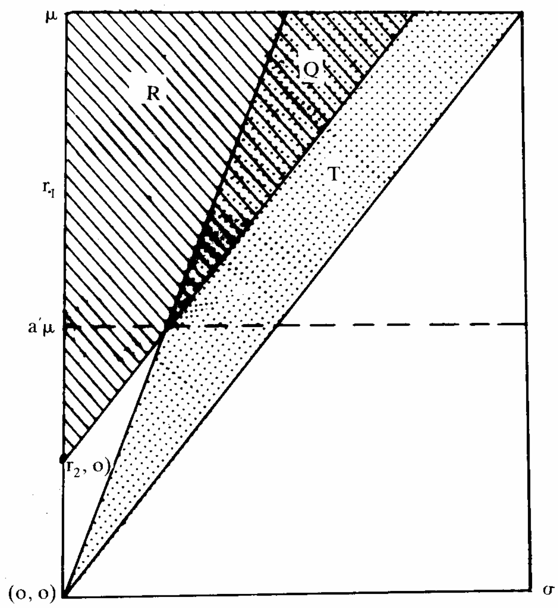

FIG. 11(b). Relative to the lower fixed rate (r_2), any profit share aμ (a' < a < 1) may yield a 'relatively superior' equity share.

In equilibrium, it is assumed that real investors are indifferent between issuing a risk-free asset (r, o), or an equity share (m_b, S_b) based on a fixed profit-sharing ratio a. This basic equity issue is translatable through diversification into an element (m_e, s_e) $\in$ E of the efficient frontier. Then, the question at any interest rate (r) and fixed profit sharing ratio (a), is whether (m_e, S_e)$\in$ Q, or not. Here again there are two possible situations described in Figure 12(a), (b) as

$$\text{(a)}\ (m_e, s_e) \notin Q$$
$$\text{(b)}\ (m_e, s_e) \in Q$$

The extent of diversifiability (g) is maintained at pre-specified level as usual, to allow for undiversifiable task.

In the first case (a), the (linear) indifference curve of the real investor combines the risk-free asset (r_1, o) and the equity share (m_b, S_b) However, at this assumed equilibrium the translated efficient value (m_e, S_e) of the basic share, is not a member of set (Q). In this case the risk-free asset (r_1, o) cannot be driven out of the market. Both debt and equity will be practiced at equilibrium.

In the second case (b), we maintain the same specified extent of diversifiability and the same profit-sharing ratio as in the previous case. However, the representative real investor, now, has a higher risk-aversion rate, judging by the steeper indifference curve. This situation yields a lower interest rate r_2 (i.e. $0 < r_2 < r_1$), such that we are left with (m_e, s_e)$\in$ Q at the assumed equilibrium. The equity share is now 'relatively superior' compared to r_2.

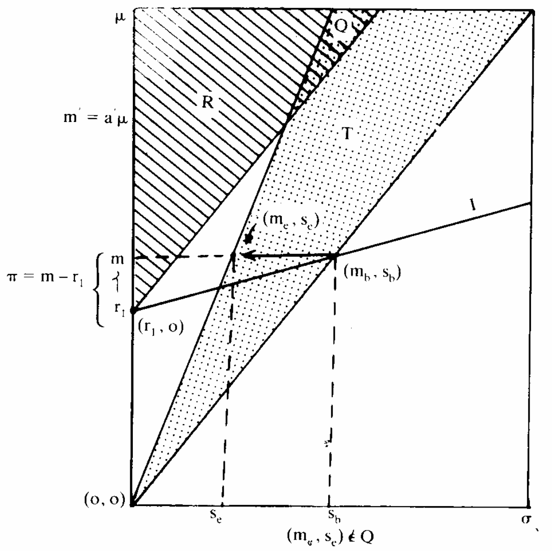

FIG. 12(a). At the given Pareto optimal equilibrium both "debt/equity" co-exist since the sharing ratio fails to yield an element in (Q). That is attainable at very high sharing ratios but it violates Pareto optimality (real investor will be worse-off).

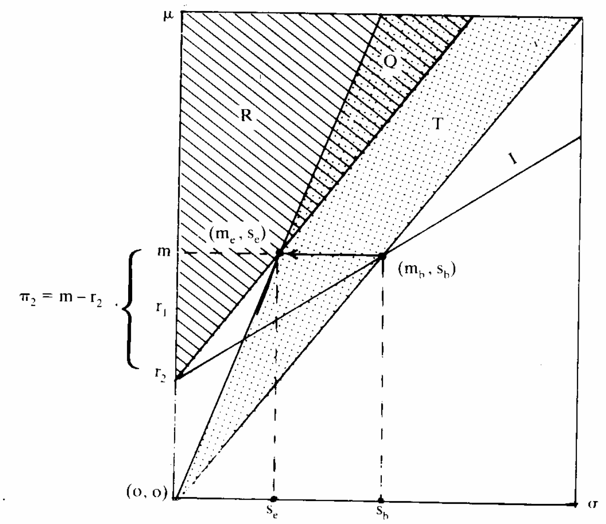

FIG. 12(b) The *ribā*-free equilibrium: Here the same sharing ratio ($m = a\mu$) yields an element in (Q) relative to a lower interest asset (r_2, o). Here Pareto optimality is maintained.

$$(m_e, s_e) \in Q \supseteq R$$

N.B. real investor has a steeper (linear) indifference curve compared to Figure 12(a).

Hence, *riba*-free Pareto optimality is already achieved in case (b) where at the assumed equilibrium the risk-free asset (r_2, o) is completely driven out of the market and all financial investors are now better off with pure equity finance. However, in case (a) the same profit-sharing ratio and the same extent of diversifiability, failed to produce such a Pareto optimality. The difference between the two cases is mainly due to difference in risk-aversion rates of the typical real investor adopted in each case. This attitudinal difference is embodied in different risk-premia, defined a:

$$\text{a)}\pi_1 = m_b - r_1 > 0$$
$$\text{b)}\pi_2 = m_b - r_2 > 0$$

where $\pi_2 > \pi_1$ since m_b is fixed and $r_2 < r_1$.

The idea is that: the more risk-averse is the real investor the lower will be the fixed interest he would promise to pay as alternative option to the fixed profit-sharing ratio. In fact, if it were not for the rationality axiom imposed on the slope of indifference curves (see p.11) the very positivity of the interest rate would be questionable!

Hence, the real investor's attitude towards risk plays a central role in our solution of the financial choice problem. In the unlikely case of risk-neutrality [perfectly horizontal (m-s) indifference curve], we have already seen in Figure 7 that:

$$\pi = m_b - r = 0$$

and no potential of diversifiability, whatsoever, can yield a situation as in case (b). But generally, the larger is the size of π, the more likely that any given extent of risk diversification would be sufficient to yield a situation as in case (b). Thus, the following result can be derived :

Result (5): Given risk-averse real investor, being initially neutralized between debt or equity issues, it is likely with any restricted extent of risk diversification that a *riba*-free Pareto optimality is attainable no matter how risk-averse are financial investors. The more risk-averse is the real investor, the more likely is the attainment of such a Pareto optimality.

2.6 Shift of Emphasis

Result (5) is perhaps the most significant finding of the present study. It asserts the possible attainment of a *riba*-free equilibrium in the efficient market, despite the explicit allowance for:

a) high risk-aversion rates of financial investors,

b) restricted diversifiability of risk, and hence persistence of undiversifiable risk.

The main significance of the above result, lies in the shift of emphasis it makes from the supply side factors to those of the demand side. Thus, what really matters is not the (limited) potentiality of risk diversification or the exceptionally high risk-aversion rates of funds suppliers. It is rather the demanders of funds attitude towards risk. The likelihood of the *riba*-free Pareto optimality is re-enforced or weakened, depending on the size of the premium π which reflects the real investor's attitude to wards risk.

Now that in real life we witness the simultaneous issuance of equity and debt, we cannot jump to the conclusion that case (a) above of Figure 2 is the appropriate description of real practice. In the first place the ideal conditions of informational efficiency are difficult to maintain in actual practice. This point is discussed in the next section.

Part-3
The Demand Side Perspective

Our point of departure has been to neutralize the representative real investor in the Miller-Modigliani sense, thus making him indifferent between issuing debt or equity. This has enabled us to focus on the supply side of the financial choice problem. The previous analysis has, however, culminated in the finding that neither high risk-aversion rates of financial investors, nor limited diversifiability of risk, are necessarily preventive to a *riba*-free Pareto optimality in the informationally-efficient market. We have, accordingly, highlighted the real investors' attitude towards risk (being reflected in the size of the premium π) as a significant demand side factor in the financial choice problem.

3.1 Why People Hold Debt Assets

Now, the question is how to derive through our simple ISB structure an explanation to the real life fact that both debt and equity co-exist as complementary financial assets. This observable fact appears consistent with the implication of Figure 12(a), where the *ex- ante* risk-premium (π_1=m_b-r_1) was not large enough to allow the attainment of *riba*-free Pareto optimality. The relatively low π_1 was an indicator of a relatively favorable attitude towards risk of real investor - as contrasted to the alter native situation in Figure 12(b).

As such, one is tempted from Figure 12(a), to confirm the commonly held belief that real investors are generally endowed with fairly favorable attitudes towards risk. This means (in terms of our ISB model) that the *ex-ante* risk-premium $\pi = m_b - r$ is sufficiently small to make debt finance more attractive to a broad range of potential financial investors.

In this sense Figure 12(a) is taken as a simple testable hypothesis that is not contradicted by factual evidence, whereas the rival, the *riba*-free Pareto optimality [of Figure 12(b)], appears to be contradicted by factual evidence.

3.2 Informational Efficiency and Testability of Hypotheses

The two situations of Figure 12 cannot be treated as simple testable hypotheses. Similar critique of testability, like what Roll (2) expressed towards the supportive empirical tests of the CAPM, is also relevant here. Roll regarded the various results of empirical tests (like those of BJS and FM) as tautological.

Similarly, here, if the assumption of an informationally efficient market was granted, the alternative situations of Figure 12(a, b) could be treated as simple testable hypotheses. But, in fact the hypothesis to test is a complex one, since the real life discrepancy from informational efficiency cannot simply be assumed away far from being testable.

3.3 Real Investors and the Information Factor

Fortunately, the simple analytical structure of our ISB is easily adaptable to depict deviations from the ideal assumption of informational efficiency. In fact, the very nature of a dynamic economy makes it impossible at any point of time to maintain the homogeneous expectations assumption. It is not only that information is costly, but more significantly it is the problem of uneven access to the flow of economic information for all interested agents. If we stick to our broad classification of agents into real investors and financial investors, then the dynamic flow of new economic information can greatly be simplified. The unrealistic assumption of homogeneous expectations will then be modified by allowing the two groups of investors have heterogeneous expectations. To maintain analytic simplicity, we shall introduce the assumption of *group-specific homogeneous expectations*. That is, real investors as a group have homogeneous expectations, which are different from those held by financial investors.

Then, taking a short term perspective of the dynamic scene, the very latest productive economic knowledge would be more directly accessible to real investors (who are involved in actual production) than to the remote suppliers of investible funds. Like the Schumpeterian entrepreneurs, the group of real investors represents the spearhead of dynamic economic growth. Dynamic change is in fact the main source which generates profits to entrepreneurs in Schumpeter's approach.[(3)] Basically, the entrepreneur is not a capitalist and he has nothing to invest. He is, however, fast enough to acquire command over another scarce economic resource, that is, the new flow of productive economic knowledge. He is, thus, having a relative informational advantage to exploit against inactive capitalists. The Schumpeterian entrepreneur is such an active self-confident person with an egoistic spirit-a set of qualities which enable him to dig out new opportunities of future economic prospects much faster than others-and to exploit this knowledge in the short run to his own best. Emergence of profits in the short run is caused by "... the higher productivity of the new method... and not the uncertainty of the results or the unpredictable nature of innovations"[(4)], a point which puts the Schumpeterian system in a direct contrast with that of Knight (1921).

Now, our homogeneous group of real investors could be regarded as entrepreneurs in the Schumpeterian sense, despite our allowance for uncertainty of results. The assumed measureability of uncertainty through the mean-variance probabilistic approach, still provides a solid basis for productive economic information (i.e. ex-ante data). Admittedly, our provision for probabilistic information is in clear contrast with Knight's unmeasurable concept of uncertainty, which he uses to explain the role of entrepreneurs, but the assumption of measureability is indispensable for developing simple models.

Thus, given the group-specific assumption of homogeneous expectations as out lined above, our simple ISB model can be used to reveal the effect of informational inefficiency in a short perspective, where the group of real investors has already acquired a relative informational advantage against financial investors.

3.4 The Consequence of Informational Inefficiency

To see how the group of real investors may capitalize on their relative informational advantage, we shall adapt the simple ISB model to represent two sets of expectations. The financial investors' expectations will continue as they used to be in the long run equilibrium. They still hold at the beginning of the period the expectation that the i.i.d. return variables have mean μ and variance σ^2. However, the group of real investors has already captured new productive information which will adapt their expectations to π' and σ^2 such that $\mu' > \mu$. The risk parameter σ is assumed unchanged whereas the mean return μ' is now higher, due to the higher expected productivity value of the new knowledge.

Clearly this Situation would result in two ISB's being visualized by each of the two groups. When the two boxes are brought together they result in the situation shown in Figure 13(a). The smaller box is the one visualized by financial investors, whereas the bigger box is the one visualized by real investors. Given a fixed profit-sharing ratio a, we shall end up with two different expectations of equity shares:

(i) $m_b = a\mu$ (visualized by financial investors)

(ii) $m'_b = a\mu'$ (visualized by real investors)

and obviously $m'_b > m_b$; see Figure 13(a).

Now, if the representative real investor is to remain neutralized as we did before, then he may either issue a new debt asset (r', o) or the old equity share which is now yielding $(m'_b > s'_b)$ -given the fixed profit-sharing ratio. Note that the new interest r' is considerably higher than the old rate r.

3.5 Real Investor's Preference

Then, the problem is how the real investor may capitalize on the relative informational advantage. Naturally, we cannot keep him neutralized as before, between is suing debt asset (r, o) or an equity with sharing ratio a. He is now in a better informational position relative to financial investors, and he may exploit that position to his own best. The fixed profit-sharing ratio, a, cannot be changed in the short run and it cannot be simply lowered to the real investor's own advantage. Hence, if the real investor has to choose between issuing the old debt asset (r, o), or the equity share with new value $(m'_b > s'_b)$, he would definitely have strict preference for the former. The old debt issue now falls on a higher indifference curve than the issue of equity with the new expected value, and hence all real investors may decide to rely exclusively on issuing the debt asset (r, o) to attract investible funds. This situation is depicted in Figure 13(a).

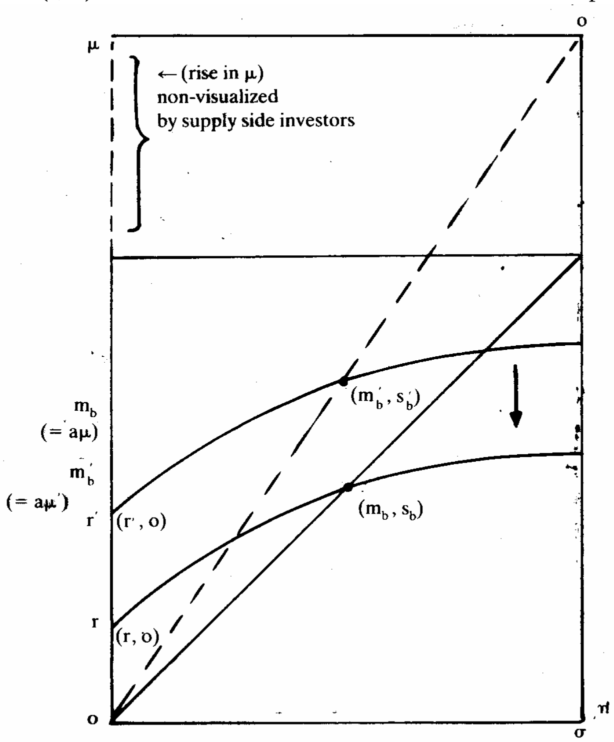

FIG. 13(a) Real investors now prefer issuance of debt asset (r, o) to equity. The enlarged box is the one visualized only by real investor.

3.6 Disturbance of a Riba-Free Equilibrium

To bring this point to sharper focus, Figure 13(b) starts from a *riba*-free long term equilibrium where the old debt asset (r, o) had already been driven out of the market. Yet, with the short term relative informational advantage they recently acquired, real investors would be far worse off if they keep issuing equity with the new value $(m_b' > s_b')$. For in this manner they would surrender larger profit shares to the funds' suppliers whose (long term) profit expectations are too modest.

Given the fixed profit-sharing ratio, and the irrelevance of the debt asset (r, o) under the *riba*-free Pareto optimality, the most direct way for real investors is to compete for investible funds by issuing *higher interest debt assets*. This process would eventually end up at a somewhat higher interest rate r'', where:

$$r < r'' < r'$$

The typical real investor is still better off with issuing such higher interest debt compared to the equity option; see Figure 13(b). On the other hand, the rise in r will cause set (R) to diminish, thus significantly disturbing the assumed *riba*-free Pareto optimality for the last period of Figure 12(b). In the eyes of funds' suppliers (who are at a relative informational *disadvantage*) the rise in the interest r has considerably reduced the previous risk-premium from $\pi = m_b - r$ to $\pi'' = m_b - r''$, thus making equity less attractive.

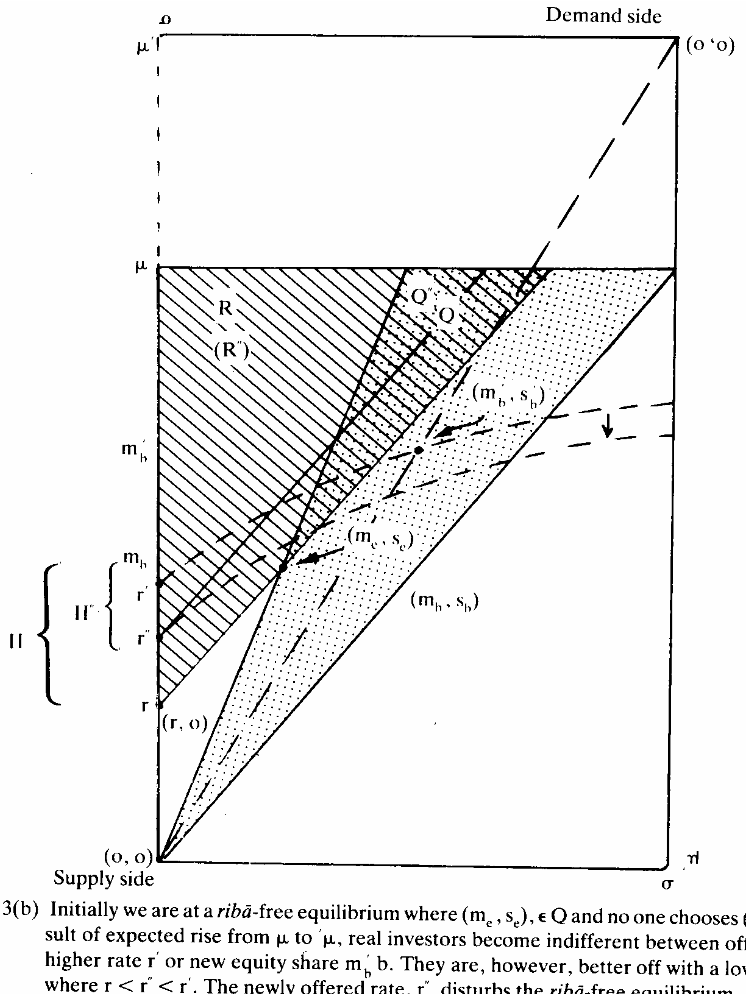

FIG. 13(b) Initially we are at a *ribā*-free equilibrium where $(m_e, s_e) \in Q$ and no one chooses (r, o). As a result of expected rise from μ to 'μ, real investors become indifferent between offering a much higher rate r' or new equity share m_b' b. They are, however, better off with a lower interest r'' where $r < r'' < r'$. The newly offered rate, r'', disturbs the *ribā*-free equilibrium, due to reduction of Q to Q'' i.e. $(m_e, s_e) \notin Q''$.

We have already seen how a short-term dynamic change has disturbed the condition of informational efficiency, giving practical advantage to the demand side real investors, also, how a previous *riba*-free Pareto optimality has been disturbed by the demand side initiative to issue debt assets. Hence, since in practice it is impossible to maintain the ideal type of a stable informationally-efficient market, it appears as though dynamic episodes of informational inefficiency are the significant forces which generate the observed debt activities. Such an interpretation for real life practices appears, by far, more appealing and less restrictive than that of Figure 12(a). This finding is now expressible in the following result:

Result (6)

The observable activity of debt finance is mostly a demand side initiative, taken by real investors who choose to issue debt assets in order to exploit a relative ex-ante in formational advantage.

This result describes the practice of debt finance as essentially a demand side choice, and a derivative of informational inefficiency. It lies in sharp contrast with the alternative approach which emphasizes the force of supply side choices in terms of risk-aversion, while neutralizing the demand side.

Part-4
Concluding Remarks

The previous discussion has revealed how an Islamic *riba*- free Pareto optimality can be maintained in an informationally efficient market, notwithstanding the counter forces of exceptionally high risk-aversion rates of funds' suppliers, and persistence of undiversifiable risk. These two counter forces are usually taken as the main objective forces which impose debt finance in actual practice, as it is perceivable from standard investment theory. Accordingly, the financial Islamization policy, which involves replacement of interest-based debt finance by profit-sharing equity finance, is often claimed as deterring to the 'natural' economic forces which generate investible funds in the most efficient way. Given the pioneering Miller-Modigliani theory which has provided grounds for believing that debt/equity ratio is not a critical parameter to the demand side, the emphasis continued to be placed on the supply side of investible funds. Thus, secular economists tend to believe that debt and equity must be allowed to freely operate in order not to retard the supply process of investible funds. Suppliers of funds are generally viewed as variably risk-averse, and hence, provision of debt assets would suit the tastes of those with higher risk-aversion rates. In this sense, if equities alone were issued, the financial system would fail to attract the funds of a significant portion bf risk-averse suppliers.

We have deliberately allowed for such counter forces of the financial Islamization policy, by over-emphasizing the risk-averse nature of suppliers. Such an attitudinal factor is further exaggerated by allowing the demand side real investors to have a relatively more favourable attitude towards risk. Persistence of undiversifiable risk has also been allowed for. Nonetheless, it was interesting to find out how a *riba*-free Pareto optimality may resist such strong counter forces, within the assumed informationally-efficient market.

Then, as a step towards realism, the ideal condition of informational efficiency has been relaxed. To provide a sharp focus for the effect of informational inefficiency, our point of departure has been a long-term *riba*-free Pareto optimality. Informational inefficiency has been introduced through a short-term dynamic change, and the analysis revealed how the previous *riba*-free Pareto optimality has been disturbed through an initiative from the demand side to issue debt assets.

It appears on this ground that the observed real life debt practices are basically triggered off by the demand side. The idea is that real investors wish to borrow whenever a lucrative profit prospect is foreseen through a dynamic change. Thus, it implies *negative attitude towards sharing* on the part of real investors (who enjoy a relative informational advantage in a continuous sequence of short-terms) rather than negative attitude towards risk on the part of investible funds suppliers.

Does the System Sympathize with Risk-Avertors?

Economic systems, generally, are structured to cater for the preferences of active well-organized agents, much more than those of passive and unorganized agents. The financial structure generated by a free market system must therefore embody the vested interests of active profit-making beneficiaries who are located at the forefront of productive economic information.

It is not surprising, therefore, that the modern financial debt system has been established through the significant role that was historically assumed by industrialists. The initial stages of the industrial revolution in Europe marked a historical turning point where large profits started to accrue to the enterprising industrialists out of technological innovations. It was not a sheer coincidence that the same historical stage of lucrative profit prospects witnessed the earlier conflicts and dialogue between the Christian Church and the industrial pioneers to legalize the debt system. That was essentially a 'demand' rather than a 'supply' side concern. This demand side historical background may be contrasted with the recent supply side success of Islamic banking in attracting massive investible funds, merely through profit-sharing schemes. As it appears, the modern debt system is mainly a reflection of demand side preferences rather than a means to accommodate the supply side taste for 'safe' returns.

Insight from Riskless Neo-Classical Theory

Our viewpoint that the debt system emerges mainly as a derivative of informational inefficiency within dynamic short-term perspective, is brought to a sharper focus through the neo-classical perfect foresight models. In principle, nothing prevents the competitive long-term zero profit equilibrium from being an Islamic *riba*-free equilibrium. The *mudarabah* firm is indeed a logical possibility that suits the long-term theoretical structure where total output is exhaustively shared between labour and capital resources in a direct profit-sharing contract. Weitzman (1984) acknowledges such a point, remarking, "The long-term allocation pattern towards which the economy tends... is independant of the compensation system," (p.96). However the Islamic *mudarabah* firm is empirically irrelevant to the Western developed economies. It is, thus the structural 'conservatism' of positive economics that precludes such a logical possibility at the theoretical long-term equilibrium [(5)].

But the crucial issue relates to the rise of pure profits in the short run to a profit-maximizing entrepreneur. This has been one of the most controversial issues in the literature; see Siddiqi (1971). The critical questions are: Why should the competitive short-term allocation pattern be dependant on the fixed interest/wage compensation system? Why should owners of productive resources (capital/labour) sell their services to a third profiteering party? If the short-term perfect foresight condition applies to all economic agents, then why should owners of productive resources be content with marginal productivity rewards which secure maximum profit to the entrepreneur? Why shouldn't they continue, as in the long-term, combining their productive services directly in a *mudarabah* share system?.

The problem is that if all agents simultaneously hold the same perfect foresight about future profits, then the ideal model cannot justify the role of the entrepreneur!. In fact this has been the central query that led Knight (1921) to develop his approach of economic uncertainty(6). However, Knight provided an unmeasurable brand of uncertainty that cannot be easily absorbed within the neoclassical models. Alternatively, to preserve the internal consistency of the neoclassical models it is essential to allow for a group-specific informational structure.

This necessitates that owners of productive resources only possess perfect foresight regarding the market wage and interest rates. But they must not possess any such foresight about future profits. The latter must be confined to the group of entrepreneurs alone. Thus, the typical entrepreneur will be capitalizing on a relative *ex-ante* informational advantage, which is the only scarce economic resource that he exploits.

In this sense, a possible long-term *riba*--free equilibrium has been disturbed by the entrepreneurial intervention to account for the fixed 'interest' compensation system. Ambitious profiteering entrepreneurs, generally, wish to borrow rather than share lucrative profit prospects with others.

Notes

1) The very concept of 'risk-premium' embodies the sense of compensation for risk-taking. For example the expected rate of return of a common stock is given by the equation of the security market line, as the sum of a risk-free rate and a risk-premium, compensating (financial) investors for taking the risk associated with the investment, Haugen (1988; p.167). But the question is this: If it makes no difference for real investors which kind of asset to issue, then, why should they be keen to compensate financial investors for taking the risk associated with equity shares? In our present context the difference:

$$\pi = m_b - r > 0$$

does act as a 'risk-premium' to financial investors, though it is originally a reflection of real investors attitude towards risk.

2) **Roll** (1977).
3) **Schumpeter** (1961), p. 64; **Siddiqi** (197l), pp. 25-33.
4) **Siddiqi** (1971), p. 33. Also, see **Schumpeter** (1961), p. 33.

5) A fundamental methodological point in positive economics is that the ideal types cannot be visualized independently of their empirical counterparts. Although they may not be

descriptively accurate, yet they must be analytically relevent to the empirical phenomena. Friedman (1966). This explains why economists do not bother to workout implications for logical structures consistent with the ideal types, but having no relevence to the observed socio-economic structure: e.g. the Islamic *mudarabah* firm. The latter is in fact a logical possibility consistent with the zero-profit long-term equilibrium in a perfectly competitive market, where capital and labour resources exhaust the total output. It is also logically consistent with the ideal condition of perfect future foresight in the short-term. Such possibilities are ignored simply due to the structural conservatism of positive economics.

6) The problem still remains if the risk element is introduced into the classical perfect foresight models, where the assumption of a profit-maximizing entrepreneur is replaced by one of expected utility maximization in the Neuman-Morgenstern sense*. In the latter case the entrepreneur is clearly defined as risk-averse, but he still enjoys a relative informational advantage in terms of the probabilistic distribution of future profit. In this case the information factor provides the only possible justification for the emergence of profit to the entrepreneur, since the latter is assumed risk-averse.

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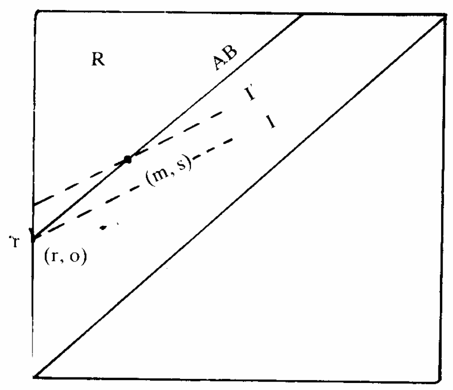
* Such an approach is demonstrated in Henderson and Quandt (1984).

Appendix (A)

Proof of Lemma (p.18)

We may take any line AB within the upper triangle of the ISB parallel to its main diagonal as in the figure below.



Then the lemma simply asserts that: if (m, s) is any point along line AB, then the rationality axiom leads to the strict preference :

$$(m, s) \rightarrow (r, o)$$

where (r, o) is the risk-free asset placed at the end of line AB.

Proof

Assuming linear (m-s) indifference curves, the proof is straight forward*. The rationality axiom (pp. 9-10), precludes the existence of any (m-s) indifference curve which is either

i) steeper than line AB, or
ii) identical to line AB

As a result the preference relations
i) (r, o) $\rightarrow$ (m, s), or
ii) (r, o) $\leftarrow$ (m, s) (for indifference)

are correspondingly precluded, implying that the only possibility is the Strict relation:

iii) (m, s) $\rightarrow$ (r, o), which obeys the axiom; (see the figure)

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* This is also true for convex indifference curves.

Appendix (B)

Given a fixed investible fund, F. for any arbitrary financial investor, and the i.i.d. assumption about the net return per unit dollar investment, X,, with E (X_i)=m and Var (X_i)=s^2, the following lemma and corollary can be established.

Lemma

The most efficient allocation is the one where the fixed investible fund, F, is evenly allocated over the full range of the n real investments.

Corollary: Any allocation $\{F_i\}$ that does not cover the full range of real investments (i.e. where F, = 0 for some i), is less efficient.

Proof

For any arbitrary allocation $\{F_i\}$ of the fixed investible fund, F, there is a set of net random returns { X, F, of the F, dollars supplied to the real investors (i=1, 2, .., n). Thus. the fund, F, yields a total random return:

$\sum_{i=1}^{n} X_i F_i$ from the n real investments to be distributed between the n real investors and the arbitrary financial investor

Then, the following results can be verified:

i) Any allocation $\{F_i\}$ yields the same expected return i.e

$$E\left(\sum_{i=1}^{n} X_i F_i\right) = m\sum_{i=1}^{n} F_i = mF$$

ii) The variance of any arbitrary allocation {F} is given by:

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i F_i\right) = \sum_{j=1}^{n}\sum_{j=1}^{n} F_i F_j \operatorname{cov}(X_i, X_2)$$
$$= \sigma^2 \sum_{i=1}^{v} F_i^2$$

Since cov (X_i, X_i) = o i $\neq$ j (due to independence)
= σ^2 i = j (as given)

iii) The even allocation F_i = (1/n) F = $\bar{F}$ (i = 1, 2, ..., n), yields the variance:

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i \bar{F}\right) = \bar{F}^2 \sum_{i=1}^{n} \mathrm{Var}(X_i)$$
$$= n\bar{F}^2\sigma^2$$

iv) Then taking the difference between the variance of any arbitrary allocation as in (ii), and that of the even allocation in (iii), we find:

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i F_i\right) = \mathrm{Var}(\overline{F}\sum_{i=1}^{n} X_i)$$
$$= \sigma^2 \sum_{i=1}^{n} (F_i - \overline{F})^2 > 0$$

Hence, the full range *even* allocation achieves highest efficiency in the sense of minimal variance. Any other allocation $\{F_i\}$, including cases where F_i=o for some i, are therefore less efficient.

سندات الدين والأسهم في سوق مالية أولية: نظريتها ودلالاتها الإسلامية

سيف الدين إبراهيم تاج الدين

وزارة التخطيط، الرياض، المملكة العربية السعودية

المستخلص. يفترض هذا البحث كراهية المخاطرة لدى مقدمي الأرصدة في بيئة تتسم بعدم التأكد، ويبرهن على إمكان تحقيق أمثلية باريتو في سوق مالية أولية خالية من الربا، كما يعلل البحث ظهور النظام المختلط للتمويل بسندات الدين والأسهم بأنه نتيجة للانحراف عن حالة الكفاءة في المعلومات.

ويبين أن المستثمر المالي الذي يواجه خطرًا غير قابل للتنويع (التشتيت)، قد يفضل اقتناء "الأسهم"على اقتناء سندات الدين العديمة المخاطرة حتى لو كان هذا المستثمر أكثر كراهية للمخاطرة من المصدر الأول للأسهم.

ويستنتج البحث أن القوى الديناميكية في سوق حرة تولد نظامًا ماليًا يحابي طالبي الأرصدة النشطين، أكثر مما يتعاطف –كما يدعي التحليل التقليدي– مع عارضي الأرصدة الكارهين للمخاطرة.

ويبدو للباحث أنه لا أساس للدعوى القائلة بأن أسلمة النظام المالي قد تؤدي إلى خسارة بعض الكفاءة في عملية عرض الأرصدة.