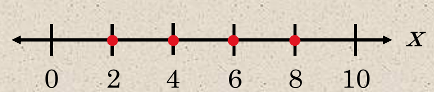
§ 4.1

Probability Distributions

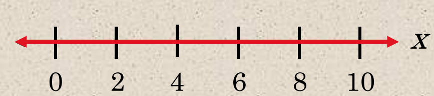
Random Variables

A **random variable *x*** represents a numerical value associated with each outcome of a probability distribution.

A random variable is **discrete** if it has a finite or countable number of possible outcomes that can be listed.



A random variable is **continuous** if it has an uncountable number or possible outcomes, represented by the intervals on a number line.



Probability Distributions

The probability distribution of a random variable is a table, graph, formula used to specify all possible values of a discrete r.v. along with their respective probability .

Random Variables

Example:

Decide if the random variable *x* is discrete or continuous.

a.) The distance your car travels on a tank of gas

The distance your car travels is a continuous random variable because it is a measurement that cannot be counted. (All measurements are continuous random variables.)

b.) The number of students in a statistics class

The number of students is a discrete random variable because it can be counted.

Discrete Probability Distributions

A **discrete probability distribution** lists each possible value the random variable can assume, together with its probability. A probability distribution must satisfy the following conditions.

In Words	*In Symbols*
1. The probability of each value of the discrete random variable is between 0 and 1, inclusive.	$0 \le P(x) \le 1$
2. The sum of all the probabilities is 1.	$\Sigma P(x) = 1$

Constructing a Discrete Probability Distribution

Guidelines

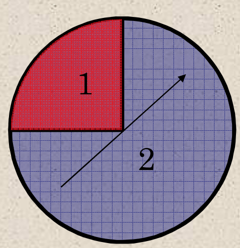
Let x be a discrete random variable with possible outcomes $x_1, x_2, \ldots, x_n$.

1. Make a frequency distribution for the possible outcomes.
2. Find the sum of the frequencies.
3. Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
4. Check that each probability is between 0 and 1 and that the sum is 1.

Constructing a Discrete Probability Distribution

Example:

The spinner below is divided into two sections. The probability of landing on the 1 is 0.25. The probability of landing on the 2 is 0.75. Let *x* be the number the spinner lands on. Construct a probability distribution for the random variable *x*.



x	$P(x)$
1	0.25
2	0.75

Each probability is between 0 and 1.

The sum of the probabilities is 1.

§ 4.2

Binomial Distributions

Binomial Experiments

A **binomial experiment** is a probability experiment that satisfies the following conditions.

1. The experiment is repeated for a fixed number of trials, where each trial is independent of other trials.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success (S) or as a failure (F).
3. The probability of a success $P(S)$ is the same for each trial.
4. The random variable x counts the number of successful trials.

Notation for Binomial Experiments

Symbol	*Description*
n	The number of times a trial is repeated.
$p = P(S)$	The probability of success in a single trial.
$q = P(F)$	The probability of failure in a single trial. ($q = 1 - p$)
x	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.

Binomial Experiments

Example:
Decide whether the experiment is a binomial experiment. If it is, specify the values of *n*, *p*, and *q*, and list the possible values of the random variable *x*. If it is not a binomial experiment, explain why.

- You randomly select a card from a deck of cards, and note if the card is an Ace. You then put the card back and repeat this process 8 times.

This is a binomial experiment. Each of the 8 selections represent an independent trial because the card is replaced before the next one is drawn. There are only two possible outcomes: either the card is an Ace or not.

$n = 8 \quad p = \frac{4}{52} = \frac{1}{13} \quad q = 1 - \frac{1}{13} = \frac{12}{13} \quad x = 0,1,2,3,4,5,6,7,8$

Binomial Experiments

Example:
Decide whether the experiment is a binomial experiment. If it is, specify the values of *n*, *p*, and *q*, and list the possible values of the random variable *x*. If it is not a binomial experiment, explain why.

- You roll a die 10 times and note the number the die lands on.

This is not a binomial experiment. While each trial (roll) is independent, there are more than two possible outcomes: 1, 2, 3, 4, 5, and 6.

Binomial Probability Formula

In a binomial experiment, the probability of exactly x successes in n trials is

$$P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}.$$

Example:

A bag contains 10 chips. 3 of the chips are red, 5 of the chips are white, and 2 of the chips are blue. Three chips are selected, with replacement. Find the probability that you select exactly one red chip.

$p =$ the probability of selecting a red chip $= \frac{3}{10} = 0.3$

$q = 1 - p = 0.7$

$n = 3$

$x = 1$

$$P(1) = {}_3C_1 (0.3)^1 (0.7)^2$$
$$= 3(0.3)(0.49)$$
$$= 0.441$$

Binomial Probability Distribution

Example:

A bag contains 10 chips. 3 of the chips are red, 5 of the chips are white, and 2 of the chips are blue. Four chips are selected, with replacement. Create a probability distribution for the number of red chips selected.

$p =$ the probability of selecting a red chip $= \frac{3}{10} = 0.3$

$q = 1 - p = 0.7$

$n = 4$

$x = 0, 1, 2, 3, 4$

x	$P(x)$
0	0.240
1	0.412
2	0.265
3	0.076
4	0.008

The binomial probability formula is used to find each probability.

Finding Probabilities

Example:
The following probability distribution represents the probability of selecting 0, 1, 2, 3, or 4 red chips when 4 chips are selected.

x	$P(x)$
0	0.24
1	0.412
2	0.265
3	0.076
4	0.008

a.) Find the probability of selecting no more than 3 red chips.

b.) Find the probability of selecting at least 1 red chip.

a.) $P(\text{no more than } 3) = P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$= 0.24 + 0.412 + 0.265 + 0.076 = 0.993$

b.) $P(\text{at least } 1) = P(x \ge 1) = 1 - \underbrace{P(0)}_{\text{Complement}} = 1 - 0.24 = 0.76$

Graphing Binomial Probabilities

Example:
The following probability distribution represents the probability of selecting 0, 1, 2, 3, or 4 red chips when 4 chips are selected. Graph the distribution using a histogram.

x	$P(x)$
0	0.24
1	0.412
2	0.265
3	0.076
4	0.008