In steady-state, the speed deviation of the generators in the power systems is zero or closely zero by the action of the Automatic Voltage Regulator (AVR). However, in transient state, the rotor swings and the terminal voltage undergoes oscillations caused by the change in rotor angle. The Power System Stabilizer (PSS) is usually added in conjunction with the AVR to help damping such oscillations by adding an additional signal that compensates for the voltage oscillations. The rotor speed is commonly fed as input to the PSS whereas its output is added as a signal to the AVR. It is a cost-effective method for enhancing stability in any power system. A lot of types of PSS structure were proposed in the literature. In this paper, H₂-norm is used to design iteratively a PID via LMI technique. Its effectiveness is shown through a comparison with two types: the first is the conventional lead-lag controller, designed with Genetic Algorithm, and the last is an H₂-norm LMI-based robust output feedback controller. Several tests were carried out namely, regulation and tracking of the terminal voltage and mechanical torque to their respective references, and a wide parameter variation. The results show the superiority of H₂-PID to damp the power system oscillations.

Keywords: Power system stabilizer, iterative PID, genetic algorithm, H₂-norm, linear matrix inequalities.

1. Introduction

The disturbances occurring in a power system induce electromechanical oscillations of the electrical generators (called also power swings). They must be effectively damped to maintain the system's stability.

Conventional Lead-Lag (CLL) Power System Stabilizer (PSS) is widely used to damp mechanical mode oscillations following any disturbance (loading) in power system utilities. Reports [1-3] have demonstrated the performance degradation during transient conditions and changes in system loading conditions. Over the past years, considerable efforts were placed on the coordinated synthesis of power system stabilizer (PSS) parameters.

In [1], a sequential eigenvalue assignment algorithm for selecting the parameters of stabilizers in power system is proposed. A major disadvantage of this method is that the sequential addition of the stabilizers will disturb the previously assigned eigenvalues. In order to avoid this undesirable effect of eigenvalue drift, several approaches for the simultaneous tuning of stabilizer parameters have been proposed [3-8]. In [8], an effective approach based on Decentralized Modal Control (DMC) technique for the selection of PSS parameters in a multimachine power system is proposed. This method requires heavy computation due to the reduction steps of the system. In [9,10], the H∞-robust techniques were employed. However, the difficulties in the selection of the weighting functions were

Ahmed Bensenouci, Electrical and Computer Engineering Department, College of Engineering, King Abdulaziz University, POBox 80204, Jeddah 21589, Saudi Arabia. bensenouci@ieee.org
1 Originally from Electrical Engineering Department, Ecole Nationale Polytechnique, 10 Avenue Hassen Badi, El-Harrach, Algiers, Algeria

Copyright © JES 2011 on-line: journals/esrgroups.org/jes
reported. Besides, the system order is as high as the plant. This gives a complex structure and might lead to limited applicability. Besides, in [10], a frequency domain approach is proposed and which is based on eigenvalues assignment and fixed-point method, an iterative algorithm, to determine the PSS gains. However, the problem of convergence of the PSS parameters has not been proved yet. In [11], GA was proposed as an optimization technique used to determine the optimum values of the PSS gains. GA was found very attractive since it is independent of the complexity of the objective function and does not require a specific structure, or derivatives, or initial function value.

Robust controllers based on the optimization of the $H_{\infty}$-norm of the transfer matrix between the system disturbance and its output, via Riccati method or Linear Matrix Inequalities (LMI) technique [12-14] have been widely applied in control theory and applications. Such controllers show robustness against disturbance but may have a large size that may give rise to complex structure and create difficulty in implementation. To overcome this difficulty, one has to reduce the size [15-18] or, as a variation, use a specific controller structure (e.g. Proportional-Integral, PI), whose parameters can be determined via minimization of the system $H_{\infty}$-norm using an optimization technique or an iterative LMI technique [19,20].

However, in most cases, a good controller should keep the system stability and acts sufficiently fast with well-damped response. Robust control gives disturbance attenuation on selected system outputs due to disturbance effects. However, $H_2$–norm control technique helps improving the transient of some system outputs following a variation in the disturbance. Fast decay, good damping and reasonable controller dynamics can be imposed in a proper region in the left complex half plane. Many control problems and design specifications have LMI formulations, in particular, $H_2$-control. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

In this paper, $H_2$-control via LMI for an iterative PID design is carried out. The goal is to find out the optimum PSS gains. The applied technique is done for a sample power system made of a synchronous generator connected to a large system via a transmission line. To show the controller effectiveness, the results were compared to two design types. The first is a PSS designed with a robust $H_2$-control using LMI technique whereas the last is a conventional lead-lag PSS type whose gains were found using genetic algorithm. The design strategy is based on a linearized model. The robustness of the design procedure is demonstrated through diverse tests including a wide range parameter changes and reference values changes (regulation and tracking).

2. Notation

The notation used throughout the paper is stated below.

Constants:

- $V_{ref}$: reference terminal voltage
- $V_t$: terminal voltage
\( V_o \) infinite bus voltage
\( e_{fd} \) equivalent excitation (field) voltage
\( e_q \) q-axis voltage behind transient reactance
\( V_f \) stabilizing transformer voltage
\( V_z \) stabilizer output
\( T_e \) energy conversion torque
\( T_m \) mechanical input
\( \delta \) torque angle
\( \omega_r \) angular velocity
\( H(s) \) transfer function of PSS
\( K_t \) regular gain voltage
\( T_A \) voltage regulator time constant
\( K_f \) stabilizing transformer gain
\( T_F \) stabilizing transformer time constant
\( K_1-K_5 \) linearized model constants
\( \tau_{d0} \) d-axis transient open circuit time constant
\( M = 2H \) inertia coefficient
\( D \) damping coefficient
\( K, T, T_1, T_2 \) Lead-Lag parameters

3. Problem formulation

The state equations of the sample power system, shown in Figure 1, can be written in the linear vector-matrix differential equation form as

\[
x = Ax + B_1w + B_2u
\]  

Where

\[
x^T = \begin{bmatrix} \omega & \delta & E_q' & E_{fd} & V_F \end{bmatrix}^T, \quad w = \begin{bmatrix} T_m & V_{ref} \end{bmatrix}^T, \quad u = V_S
\]  

\( x \) is the state vector, \( w \) is the control vector, and \( u \) represents the PSS signal \( V_s \). \( A \), \( B_1 \), \( B_2 \) are constant matrices defined as:

\[
A = \begin{bmatrix}
\begin{array}{ccccc}
D & K_1 & K_2 & 0 & 0 \\
-\frac{K_1}{K_2} & \frac{K_1}{M} & 0 & 0 & 0 \\
0 & -\frac{\tau_{d0}}{K_4} & \frac{\tau_{d0}}{K_4} & \frac{\tau_{d0}}{K_4} & \frac{\tau_{d0}}{K_4} \\
0 & -\frac{K_AK_5}{T_A} & \frac{K_AK_6}{T_A} & \frac{1}{T_A} & \frac{1}{T_A} \\
0 & \frac{K_FK_AK_5}{T_FT_A} & \frac{K_FK_AK_6}{T_FT_A} & \frac{K_F}{T_FT_A} & \frac{K_F}{T_FT_A} + \frac{K_FK_A}{T_FT_A} \\
\end{array}
\end{bmatrix}, \quad B = \begin{bmatrix}
\begin{array}{c}
1 \\
M \\
0 \\
0 \\
0 \\
0 \\
\end{array}
\end{bmatrix}
\]

4. Lead-Lag PSS with GA (LLG)

The transfer function of the PSS, \( H(s) \) is usually of the lead-lag type as
\[ H(s) = \left[ \frac{sT}{1 + sT} \right] K \left[ \frac{1 + sT_1}{1 + sT_2} \right]^2 \]

The first term on the left is the washout term. \( K \) is the stabilizer gain and \((1 + sT_1)^2 /(1 + sT_2)^2\) represents the lead-lag terms. The washout time constant \( T \) and the denominator time constant \( T_2 \), in the stabilizer transfer function, are usually chosen to be 10 seconds and 0.05 second \([11]\), respectively. The stabilizer gain \( K \) and \( T_i \) were selected such that the dynamic stability of the power system can be improved optimally using found using GA \([11]\).

To have some degree of relative stability and to improve the settling time and damping ratio, the parameters of the PSS, \( K \) and \( T_i \), have been selected to minimize the following objective function:

\[ J = r_1 (Re(\lambda_d) + \lambda_{do})^2 + r_2 (|Im(\lambda_d)| - \lambda_{di})^2 \]

Where

\[ \lambda_d = \lambda_{do} \pm i\lambda_{di} \]

is the desired pair of eigenvalues which are selected, in this study, to be \( \lambda_d = -2 \pm j1 \), and \( r_1, r_2 \) weighting factors. \( Re/Im \) is the real/imaginary parts of the dominant eigenvalues \( \lambda_{do} \). This will shift the dominant complex poles of the closed-loop system farther to the left of the imaginary axis beyond the desired location defined by \( s = -2 \pm j1 \) in Laplace domain.

The approach used employs GA to solve this minimization problem and searches for the optimum or near optimum set of PSS parameters \( (K, T_i) \).
Fig. 1. One machine infinite bus power system

Note that the variables represent the incremental change around their nominal operating value. Three PSS types, described next, were designed to damp the mechanical oscillations mentioned above. The PSS has the generator speed as input and the signal $V_s$ as output. $V_s$ is added to the reference terminal voltage value $V_{ref}$.

5. Robust H_2 Controller (ROB)

In a typical H_2 design problem, the nominal plant model, represented by its transfer function $G(s)$, is usually known and the design problem for an output feedback control is formulated as a standard H2 problem, as described by the block diagram of Fig. 2.
Fig. 2. Output feedback block diagram

In Fig. 2, \( P(s) \) represents the plant and \( K(s) \) the controller transfer functions in Laplace domain. The controller \( K(s) \) is to be determined using \( H_2 \) design technique via LMI. In the block diagram, \( w \) represents the external disturbances \( [T_m, V_{ref}] \) (desired mechanical torque and terminal voltage), \( z = V_i \) (terminal voltage) the regulated output and \( y = \omega_i \) (generator speed) the measured outputs. The control input vector \( u = V_s \) (PSS output signal).

Let

\[
\begin{align*}
\dot{x} &= Ax + B_1 \dot{w} + B_2 u \\
P(s): \begin{cases} 
  z &= C_1 x + D_1 \dot{w} + D_2 u \\
  y &= C_2 x + D_2 \dot{w} + D_2 u
\end{cases} \\
K(s): \begin{cases} 
  \dot{z} &= A_K z + B_K y \\
  u &= C_K \dot{z} + D_K y
\end{cases}
\end{align*}
\tag{3}
\]

be the state-space realizations of the plant and the controller, respectively.

The corresponding closed-loop state-space equations \([8,9]\) are

\[
\begin{align*}
\dot{x}_{CL} &= A_{CL} x_{CL} + B_{CL} w \\
2 &= C_{CL} x_{CL} + D_{CL} w
\end{align*}
\tag{5}
\]

Where;

\[
x_{CL} = \begin{bmatrix} x \\ \xi \end{bmatrix}, \quad A_{CL} = \begin{bmatrix}
A + B_2 D_K C_2 & B_2 C_K \\
B_K C_2 & A_K
\end{bmatrix}, \quad B_{CL} = \begin{bmatrix}
B_1 + B_2 D_K D_{21} \\
B_K D_{21}
\end{bmatrix}
\]

\[
C_{CL} = \begin{bmatrix}
C_1 + D_{12} D_K C_2 & D_{12} C_K
\end{bmatrix}, \quad D_{CL} = D_{11} + D_{12} D_K D_{21}
\]

The objective is to design a controller \( K(s) \) such that the closed loop system formed by \( P(s) \) and \( K(s) \) is internally stable and the \( H_2 \)-norm of its transfer matrix from \( w \) to \( z \):

\[
\|G(s)\|_2 \leq \|C_{CL} (sI - A_{CL})^{-1} B_{CL} + D_{CL}\|_2 < \gamma
\tag{6}
\]

is solved using Linear Matrix Inequalities (LMI) technique \([10,11]\).

An optimal \( H_2 \) control design can be achieved by minimizing the guaranteed robust performance index \( \|G_{sw}\|_2 \) subject to the constraints given by the matrix inequalities (6). The MATLAB LMI control toolbox \([20]\) provides a function “hinfsyn” and selecting \( H_2 \) for this purpose. This function returns the controller parameters \( K(s) \) together with the optimal robust performance index \( \gamma \). The obtained controller is dynamic with the same order as that of the plant \( P(s) \) model and hence very large in general.

6. Iterative PID with H2 (PID)

The design problem of PID controller under \( H_2 \) performance specification is investigated, first, by studying the static output feedback (SOF) case and then extending the result to the PID case. Figure 3 shows the block diagram.
Consider the system [19, 20]:

\[
\begin{align*}
\dot{x} &= Ax + B_1w + B_2u \\
P(s) &= \begin{cases} 
    y_s &= C_s x \\
    y_r &= C_r x 
\end{cases} 
\end{align*}
\]  

(7)

Assuming that \(A\) is stable then for the system closed-loop transfer function and using the classical result within Lyapunov approach gives

\[
|G_{w,y_r}|_2^2 = \text{Trace}(C_rPC_r^T) 
\]

(8)

The \(H_2\)-performance index, for system (7) can be achieved by a SOF controller if the matrix inequalities:

\[
\begin{align*}
    \text{trace} (C_rPC_r^T) &< \gamma^2 \\
    AP + PA^T - PC_s^T C_s P + (B_2^F + PC_s^T)(B_2^F + PC_s^T)^T + B_1B_1^T &< 0 \\
    P &> 0 
\end{align*}
\]

(9)

have solutions for \((P,F)\).

The PID design with \(H_2\) specifications converts to a SOF control for the dynamics of the newly obtained system [19]:

\[
\begin{align*}
\dot{z} &= \bar{A}z + \bar{B}_1w + \bar{B}_2u \\
\bar{y} &= \bar{C}_sz \\
\bar{y}_r &= \bar{C}_rz \\
u &= \bar{F}\bar{y}_s
\end{align*}
\]

(10)

Where

\[
\begin{align*}
\bar{A} = \begin{bmatrix} A & 0 \\ C_s & 0 \end{bmatrix} & \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} & \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \\
\bar{C}_{s1} = \begin{bmatrix} C_s \\ 0 \end{bmatrix} & \quad \bar{C}_{s2} = \begin{bmatrix} 0 \\ I \end{bmatrix} & \quad \bar{C}_{s3} = \begin{bmatrix} C_s \\ 0 \end{bmatrix} & \quad \bar{C}_s = \begin{bmatrix} c_{s1}^T & C_{s2}^T & C_{s3}^T \end{bmatrix}^T & \quad \bar{C}_r = \begin{bmatrix} C_r \\ 0 \end{bmatrix}
\end{align*}
\]
Thus, once the feedback matrices $\bar{F} = \begin{bmatrix} \bar{F}_1 & \bar{F}_2 & \bar{F}_3 \end{bmatrix}^T$ are obtained using the following iterative LMI algorithm for solving the $H_2$-SOF control, the original PID gains $F = \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix}^T$ can be recovered from (9). In the following algorithm [19], use

$$A = \Delta A, B_1 = \Delta B_1, B_2 = \Delta B_2, C_s = \Delta C_s, C_r = \Delta C_r, F = \Delta F$$

**Static Output Feedback (SOF)-H2 Algorithm**

**Step 0:** Form the system state space realization:

$(A, B_1, B_2, C_s, C_r)$ and select the performance index

**Step 1:** Choose $Q_0 > 0$ and solve $P$ for the Riccati equation:

$$AP + PA^T - PC_s^T C_s P + Q_0 = 0, \quad P > 0$$

Set $i = 1$ and $X = P$

**Step 2:** Solve the following optimization problem for $P$, $F$, and $\Delta$.

**OP1:** Minimize $\alpha$ subject to the following LMI constraints

$$\begin{bmatrix} \sum \begin{bmatrix} B_2 F + PC_s^T C_s P + Q_0 \end{bmatrix} & (B_2 F + PC_s^T C_s P)^T \\ (B_2 F + PC_s^T C_s P)^T & -I \end{bmatrix} < 0 \\
\end{bmatrix}^{T} \begin{bmatrix} \text{trace}(C_r P C_r^T) < \gamma^2 \\
p > 0 \end{bmatrix}$$

Where

$$\sum = AP + PA^T + B_1 B_1^T - X C_s^T C_s P - PC_s^T C_s X + X C_s^T C_s X - \alpha P$$

Denote by $\alpha^*$ the minimized value of $\alpha$.

**Step 3:** If $\alpha^* \leq 0$, the matrix pair $(P, F)$ solves the problem. Stop.

Otherwise go to Step 4.

**Step 4:** Solve the following optimization problem for $P$ and $F$.

**OP2:** Minimize $\text{trace} (P)$ subject to LMI constraints (11) with $\alpha = \alpha^*$.

Denote by $P^*$ the optimal $P$.

**Step 5:** If $\|XB-PB\| < \epsilon$, where $\epsilon$ is a prescribed tolerance, go to Step 6;

Otherwise set $i = i + 1, X = P^*$, go to Step 2.

**Step 6:** It cannot be decided by this algorithm whether the problem is solvable. Stop.

### 7. Simulation Results

Consider a single machine-infinite bus power system whose linearized model including the voltage regulator and exciter can be represented by a block diagram shown in Fig 1 [1]. The parameters of the system are given in Table 1 [1]. The closed-loop eigenvalues for each of the three controllers are shown in Table 2. The parameters of the designed PSS are:
LLG [11]: $K=11.748 \& T_f=0.128$

**PID**: $F = \begin{bmatrix} K_p & K_I & K_D \end{bmatrix} = \begin{bmatrix} 4.01 & 0.26 & 11.7 \end{bmatrix}$

$H_{2\text{opt}} = 0.69$

**ROB**: $H_{2\text{opt}} = 1.2805 \begin{bmatrix} 431 & 2468 & 5.2e5 & 3.4e5 \\ 3968 & 22769 & 4.8e6 & 3.2e6 \\ 231 & 1387 & 3e5 & 2e5 \\ 6961 & 41173 & 8.8e6 & 5.7e6 \\ 1.3e5 & 3.3e6 & 7.7e5 & 1.6e8 & 1.1e8 \end{bmatrix}$ \begin{bmatrix} 14721 \\ 1.4e5 \\ 7132 \\ 39372 \\ 4.5e5 \end{bmatrix}$

$J = \begin{bmatrix} 3999 & -1.5e5 & 23646 & 5e6 & -3.3e6 \end{bmatrix}$

$D_k = 1.37e-5$

Several tests were done and described next.

7.1. Regulation

Figure 4 shows the dynamic responses of $T_e$, $\omega_r$, $u$, and $V_i$ when the system subjected to a 10% step increase then by 10% decrease in $T_m$ for the system equipped with each of the proposed controllers: ROB, LLG, PID. It is clear that the system equipped with PID shows best response; much less oscillations and low over-undershoots in the torque $T_e$ and speed $\omega_r$. In $V_i$-response, reduced numbers of oscillations is shown but with a maximum of 5% undershoot/overshoot (acceptable). Lowest settling time is also noticed for the PID. However, high control effort is shown in the control input "u" for the PID case.

![Electric Torque $T_e$](image1)

![Rotor speed $\omega_r$](image2)

(a) Electric Torque $T_e$  
(b) Rotor speed $\omega_r$
Fig. 4. System response following $\Delta T_m = +10\%$ then $\Delta T_m = -10\%$ (torque regulation)

Figure 5 shows the dynamic responses of $T_e$, $\omega_r$, $u$, and $V_t$ when the system subjected to a 10% step increase then a 10% decrease in $V_{ref}$ for the system equipped with each of the proposed controllers: ROB, LLG, PID. Similar remarks can apply, i.e., The system with the PID controller shows much less oscillations, lower over-undershoots in the torque $T_e$ and speed $\omega_r$, and lowest settling time. Similarly, the $V_t$-response shows reduced numbers of oscillations but with a maximum of around 5% undershoot/overshoot (acceptable). Higher control effort is also shown in the control input "u" for the PID case.
Fig. 5. System response following $\Delta V_{\text{ref}}=+10\%$ then $\Delta V_{\text{ref}}=-10\%$ (voltage regulation)

As a summary, from the tests done, the system equipped with the proposed three PSS types; LLG, ROB, PID, has shown stability whereas the PID designed with $H_2$-norm shows the best response over the other two. The electromechanical modes were well damped compared to LLG and ROB.

7.2. Tracking the reference values

Figure 6 shows the dynamic responses of $T_e$, $\omega_1$, $u$, and $V_i$ when the system subjected to a 5% ramp increase then remains constant then a ramp decrease for 10 seconds and finally remains in nominal value in $T_m$ and $V_{\text{ref}}$ for the system equipped with each of the proposed controllers: ROB, LLG, PID. Each interval lasts for 10 seconds.
It is clear that the system equipped with PID shows good response; much less oscillations but with relatively higher over-undershoots (negligible) in the speed \( \omega_r \). In \( V_t \)-response, excellent tracking is shown. Small control effort is also shown in the control input "u" for the PID case.

Figure 7 shows the system response due to the same tracking behavior in \( T_m \) only. Figure 8 shows the system response due to the same tracking behavior in \( V_{ref} \) only. It can be seen from the figures that the system is stable and the PID with \( H_2 \)-design shows the best response over LLG designed using GA and ROB.

As a summary, from the tests done, the system equipped with the proposed three PSS types; LLG, ROB, PID, has shown stability and for the PID designed with \( H_2 \)-norm shows the best response over the other two. The electromechanical modes were well damped as compared to LLG and ROB.

7.3. Parameters variation

Figure 7 shows the dynamic responses of \( T_e, \omega_r, u, \) and \( V_t \) when the system is subjected to parameters increase (\( M \) and \( \tau_{dm} \)) by 50% with an increase by 5% in both \( T_m \) and \( V_{ref} \). The system is equipped with each of the proposed controllers: ROB, LLG, and PID.

It can be seen from Figure 7 that the system is stable and the PID with \( H_2 \)-design shows the best response over LLG and ROB with only one oscillation and fast settling time. Both LLG and ROB show oscillations lightly damped and higher overshoots. The worse one is ROB.
The system with a $H_2$-based PID shows the best effective effect on the mechanical components ($\omega_r$ and $T_e$) but acceptable level in the electrical component $V_e$. Higher control effect is found in PID.

8. Conclusion

In this paper, three methods were used to design a Power System Stabilizer (PSS), in conjunction with an IEEE type AVR, for a sample power system composed of a synchronous generator connected to an infinite bus via a transmission line. The first design method, $H_2$-norm is used iteratively is designed a PID via LMI technique (PID) whereas the second is a conventional Lead-Lag is designed with Genetic algorithm (LLG), and the last is an $H_2$-norm LMI-based Robust output controller (ROB).

Several tests were carried out namely, regulation and tracking of the terminal voltage and mechanical torque to their respective references, and a wide range parameters variation. The results show the superiority of the PID to damp the power system oscillations.

This work can be extended for a multimachine power system and will include nonlinear effects on the responses. Besides, adaptation to system parameters variation will be looked upon.

(a) Electric Torque $T_e$

(b) Rotor speed $\omega_r$
Fig. 7. System response following parameters change (ΔM=Δτ_d,+=+50% and ΔT_m=ΔV_ref=+5%)

Acknowledgment

The author wishes to thank Prof. A.M. Abdel Ghany, Ph.D., MIEEE, Department of Electrical Engineering, Helwan University at Helwan, Helwan, Cairo, EGYPT, for his valuable advices.

References


