

Research Article

Spectral Shifted Jacobi Tau and Collocation Methods for Solving Fifth-Order Boundary Value Problems

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We have presented an efficient spectral algorithm based on shifted Jacobi tau method of linear fifth-order two-point boundary value problems (BVPs). An approach that is implementing the shifted Jacobi tau method in combination with the shifted Jacobi collocation technique is introduced for the numerical solution of fifth-order differential equations with variable coefficients. The main characteristic behind this approach is that it reduces such problems to those of solving a system of algebraic equations which greatly simplify the problem. Shifted Jacobi collocation method is developed for solving nonlinear fifth-order BVPs. Numerical examples are performed to show the validity and applicability of the techniques. A comparison has been made with the existing results. The method is easy to implement and gives very accurate results.

1. Introduction

The solutions of fifth-order BVPs have been the subject of active research. These problems generally arise in mathematical modeling of viscoelastic flows, physics, engineering, and other disciplines, (see, e.g., [1–4]). Agarwal's book [5] contains some theorems that discuss the conditions for existence and uniqueness of the solutions of fifth-order BVPs in detail.

Recently, various powerful mathematical methods such as the sixth-degree B-spline [6], Adomian decomposition method [7], nonpolynomial sextic spline functions [8–12], local polynomial regression [13], and others [14, 15] have been proposed to obtain exact and approximate analytic solutions for linear and nonlinear problems.

Spectral methods (see, e.g., [16–18]) provide a computational approach that has achieved substantial popularity over the last four decades. The main advantage of these methods