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A quadrature tau method for fractional differential equations with variable coefficients

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ABSTRACT

In this article, we develop a direct solution technique for solving multi-order fractional differential equations (FDEs) with variable coefficients using a quadrature shifted Legendre tau (Q-SLT) method. The spatial approximation is based on shifted Legendre polynomials. A new formula expressing explicitly any fractional-order derivatives of shifted Legendre polynomials of any degree in terms of shifted Legendre polynomials themselves is proved. Extension of the tau method for FDEs with variable coefficients is treated using the shifted Legendre–Gauss–Lobatto quadrature. Numerical results are given to confirm the reliability of the proposed method for some FDEs with variable coefficients.

1. Introduction

During the past decades, the field of fractional differential equations has attracted the interest of researchers in several areas including physics, chemistry, engineering and even finance and social sciences [1,2]. Besides numerical approaches, some approximate methods such as Adomian decomposition method [3,4], homotopy perturbation method [5], variational iteration method [6] and homotopy analysis method [7] are relatively new approaches to provide an analytical approximation to FDEs.

Spectral methods provide a computational approach that has achieved substantial popularity over the last four decades. They have gained new popularity in automatic computations for a wide class of physical problems in fluid and heat flow. Their fascinating merit is the high accuracy. So, they have been applied successfully to numerical simulations of many problems in science and engineering, see [8–12]. Recently, Esmaeili and Shamsi [13] introduced a direct solution technique for obtaining the spectral solution of a special family of fractional initial value problems using a pseudo-spectral method. An extension of spectral methods for numerical solutions of some fractional differential equations are given in [14–16]. Moreover, Doha et al. [17] introduced a new efficient Chebyshev spectral algorithms for solving linear and nonlinear multi-term fractional orders differential equations. In fact, Doha et al. [18] used a quadrature Jacobi dual-Petrov–Galerkin method for solving some ODEs. In the present paper, we construct the solution using the Q-SLT approach. This approach is based on the pseudo-spectral and tau techniques. To the best of the authors' knowledge, such approach has not been employed for solving fractional differential equations.

The fundamental goal of this paper is to propose a suitable way to approximate multi-term FDEs with variable coefficients using a quadrature shifted Legendre tau approach. This approach extends the tau method for FDEs with variable coefficients by approximating the weighted inner products in the tau method by using the shifted Legendre–Gauss–Lobatto quadrature. This technique requires a formula for fractional-order derivatives of shifted Legendre polynomials of any degree in terms of

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