<u>Differential equations of mass transfer</u>

<u>Definition:</u>

The differential equations of mass transfer are general equations describing mass transfer in all directions and at all conditions.

<u>How is the differential equation obtained?</u>

The differential equation for mass transfer is obtained by applying the law of conservation of mass (mass balance) to a differential control volume representing the system.

The resulting equation is called the continuity equation and takes two forms:

(1) Total continuity equation [in – out = accumulation] (this equation is obtained if we applied the law of conservation of mass on the total mass of the system)

(2) Component continuity equation[in – out + generation – consumption = accumulation] (this equation is obtained if we applied the law of conservation of mass to an individual component)

(1) <u>Total continuity equation</u>

Consider the control volume, $\Delta x\ \Delta y\ \Delta z$ (Fig. 1)

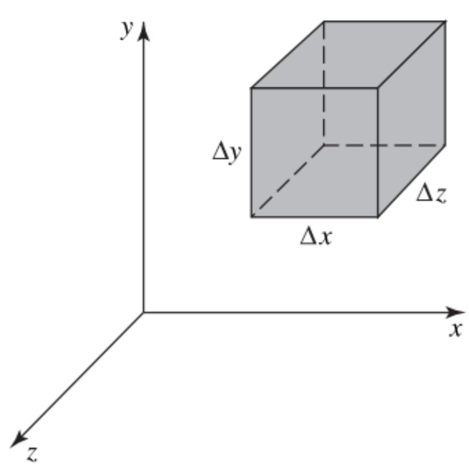


Fig. 1

Apply the law of conservation of mass on this control volume [in – out = accumulation]

direction	in	out	in - out
x	$\rho v_x \, \Delta y \Delta z \,\big|_x$	$\rho v_x \, \Delta y \Delta z \,\big|_{x+\Delta x}$	$\left(\rho v_x \big|_x - \rho v_x \big|_{x+\Delta x}\right) \Delta y \Delta z$
y	$\rho v_y \, \Delta x \Delta z \,\big|_y$	$\rho v_y \, \Delta x \Delta z \,\big|_{y+\Delta y}$	$\left(\rho v_y \big|_y - \rho v_y \big|_{y+\Delta y}\right) \Delta x \Delta z$
z	$\rho v_z \, \Delta x \Delta y \,\big|_z$	$\rho v_z \, \Delta x \Delta y \,\big|_{z+\Delta z}$	$\left(\rho v_z \big|_z - \rho v_z \big|_{z+\Delta z}\right) \Delta x \Delta y$

Accumulation = $\frac{\partial m}{\partial t} = \frac{\partial \rho \Delta x \Delta y \Delta z}{\partial t} = \Delta x \Delta y \Delta z \frac{\partial \boldsymbol{\rho}}{\partial t}$

Write the above terms in the overall equation [in – out = accumulation (rate of change)]

$$\left(\rho v_x \big|_x - \rho v_x \big|_{x+\Delta x}\right) \Delta y \Delta z + \left(\rho v_y \big|_y - \rho v_y \big|_{y+\Delta y}\right) \Delta x \Delta z + \left(\rho v_z \big|_z - \rho v_z \big|_{z+\Delta z}\right) \Delta x \Delta y = \Delta x \Delta y \Delta z \frac{\partial \boldsymbol{\rho}}{\partial t}$$

Dividing each term in the above equation by $\Delta x \Delta y \Delta z$:

$$\frac{\left(\rho v_x \big|_x - \rho v_x \big|_{x+\Delta x}\right)}{\Delta x} + \frac{\left(\rho v_y \big|_y - \rho v_y \big|_{y+\Delta y}\right)}{\Delta y} + \frac{\left(\rho v_z \big|_z - \rho v_z \big|_{z+\Delta z}\right)}{\Delta z} = \frac{\partial \boldsymbol{\rho}}{\partial t}$$

Take the limit as Δx, Δy, and Δz approach zero:

$$-\frac{\partial}{\partial x} \rho v_x - \frac{\partial}{\partial y} \rho v_y - \frac{\partial}{\partial z} \rho v_z = \frac{\partial \rho}{\partial t}$$

$$\therefore \frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z + \frac{\partial \rho}{\partial t} = 0$$

The above equation is the general total continuity equation (the velocity distribution can be obtained from this equation)

It can be written in the following form (this form can be used in all coordination system):

$$\nabla . \overrightarrow{\rho v} + \frac{\partial \rho}{\partial t} = 0$$

Special forms of the general continuity equation:

a. Steady state conditions

$$\nabla . \overrightarrow{\rho v} = 0$$

b. Constant density (either steady state or not)

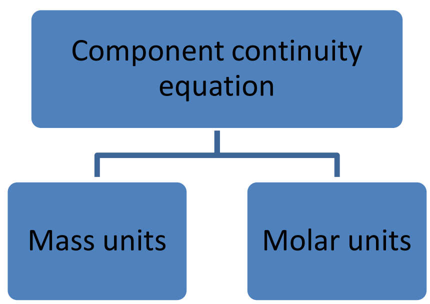
$$\nabla . \vec{v} = 0$$

c. Steady state, one dimensional flow (assume in x direction) and constant density

$$\frac{d v_x}{dx} = 0$$

(2) Component continuity equation

The component continuity equation takes two forms depending on the units of concentration; (i) mass continuity equation and (ii) molar continuity equation.



What is the importance of the component differential equation of mass transfer?

It is used to get (describe) the concentration profiles, the flux or other parameters of engineering interest within a diffusing system.

(i) Component mass continuity equation:

Consider the control volume, Δx Δy Δz (Fig. 1)

Apply the law of conservation of mass to this control volume:

[in – out + generation – consumption = accumulation]

Note: in the total continuity equation there is no generation or consumption terms

direction	in	out	in - out
x	$n_{A,x}\ \Delta y \Delta z \big\|_x$	$n_{A,x}\ \Delta y \Delta z \big\|_{x+\Delta x}$	$\left(n_{A,x} \big\|_x - n_{A,x} \big\|_{x+\Delta x}\right) \Delta y \Delta z$
y	$n_{A,y}\ \Delta x \Delta z \big\|_y$	$n_{A,y}\ \Delta x \Delta z \big\|_{y+\Delta y}$	$\left(n_{A,y} \big\|_y - n_{A,y} \big\|_{y+\Delta y}\right) \Delta x \Delta z$
z	$n_{A,z}\ \Delta x \Delta y \big\|_z$	$n_{A,z}\ \Delta x \Delta y \big\|_{z+\Delta z}$	$\left(n_{A,z} \big\|_z - n_{A,z} \big\|_{z+\Delta z}\right) \Delta x \Delta y$

Accumulation = $\frac{\partial m}{\partial t} = \frac{\partial \boldsymbol{\rho_A} \Delta x \Delta y \Delta z}{\partial t} = \Delta x \Delta y \Delta z \frac{\partial \boldsymbol{\rho_A}}{\partial t}$

If A is produced within the control volume by a chemical reaction at a rate r_A (mass/(volume)(time)

Rate of production of A (generation) $= r_A\ \Delta x \Delta y \Delta z$

Put all terms in the equation: in – out + generation – consumption = accumulation

$$\left(n_{A,x} \big|_x - n_{A,x} \big|_{x+\Delta x}\right) \Delta y \Delta z + \left(n_{A,y} \big|_y - n_{A,y} \big|_{y+\Delta y}\right) \Delta x \Delta z + \left(n_{A,z} \big|_z - n_{A,z} \big|_{z+\Delta z}\right) \Delta x \Delta y + r_A\, \Delta x \Delta y \Delta z = \Delta x \Delta y \Delta z \frac{\partial \rho_A}{\partial t}$$

Dividing each term in the above equation by $\Delta x \Delta y \Delta z$:

$$\frac{\left(n_{A,x} \big|_x - n_{A,x} \big|_{x+\Delta x}\right)}{\Delta x} + \frac{\left(n_{A,y} \big|_y - n_{A,y} \big|_{y+\Delta y}\right)}{\Delta y} + \frac{\left(n_{A,z} \big|_z - n_{A,z} \big|_{z+\Delta z}\right)}{\Delta z} + r_A = \frac{\partial \rho_A}{\partial t}$$

Take the limit as Δx, Δy, and Δz approach zero:

$$-\frac{\partial}{\partial x} n_{A,x} - \frac{\partial}{\partial y} n_{A,y} - \frac{\partial}{\partial z} n_{A,z} + r_A = \frac{\partial \rho_A}{\partial t}$$

$$\boxed{\frac{\partial}{\partial x} n_{A,x} + \frac{\partial}{\partial y} n_{A,y} + \frac{\partial}{\partial z} n_{A,z} + \frac{\partial \rho_A}{\partial t} - r_A = 0} \qquad (1)$$

Equation 1 is the component mass continuity equation and it can be written in the form:

$$\boxed{\nabla . \vec{n}_A + \frac{\partial \rho_A}{\partial t} - r_A = 0} \qquad (2)$$

(The above equation can be written in different coordinate systems since it is written in a vector form)

But from Fick's law

$$n_A = -\rho D_{AB} \nabla \omega_A + \rho_A \mathrm{v}$$

Substitute in equation 2 by this value we can get the equation:

$$-\nabla . \rho D_{AB} \nabla \omega_A + \nabla . \rho_A \mathrm{v} + \frac{\partial \rho_A}{\partial t} - r_A = 0 \qquad (3)$$

Equation 3 is a general equation used to describe concentration profiles (in mass basis) within a diffusing system.

(ii) Component molar continuity equation

Equations 1, 2 and 3 can be written in the form of molar units to get the component continuity equation in molar basis by replacing:

n_A *by* N_A; ρ *by* c; ω_A *by* y_A ; ρ_A *by* c_A *and* r_A *by* R_A

The different forms of the component molar continuity equation:

$$\frac{\partial}{\partial x} N_{A,x} + \frac{\partial}{\partial y} N_{A,y} + \frac{\partial}{\partial z} N_{A,z} + \frac{\partial c_A}{\partial t} - R_A = 0 \qquad (4)$$

Equation 4 is the component molar continuity equation and it can be written in the form:

$$\nabla . \vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0 \qquad (5)$$

But from Fick's law

$$N_A = -c D_{AB} \nabla y_A + c_A \mathrm{V}$$

Substitute in equation 5 by this value we can get the equation:

$$-\nabla . c D_{AB} \nabla y_A + \nabla . c_A \mathrm{V} + \frac{\partial c_A}{\partial t} - R_A = 0 \qquad (6)$$

Equation 6 is a general equation used to describe concentration profiles (in molar basis) within a diffusing system.

Note: you may be given the general form and asked to apply specific conditions to get a special form of the differential equation.

- **Special forms of the component continuity equation**

1. If the density and diffusion coefficient are constant (assumed to be constant)

For mass concentration equation 3 becomes

$$-D_{AB}\nabla^2\rho_A + \rho_A\nabla.\mathrm{v} + \mathrm{v}\,\nabla.\rho_A + \frac{\partial\rho_A}{\partial t} - r_A = 0$$

For constant density $\nabla.\mathrm{v} = 0$ [from the total continuity equation] (page 3 no. b)

$$\therefore\ -D_{AB}\nabla^2\rho_A + \mathrm{v}\,\nabla.\rho_A + \frac{\partial\rho_A}{\partial t} - r_A = 0$$

and for molar concentration equation 6 becomes

$$\therefore\ -D_{AB}\nabla^2 c_A + \mathrm{V}\,\nabla.c_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

2. If there is no consumption or generation term and the density and diffusion coefficient are assumed constant

For mass concentration:

$$\therefore\ -D_{AB}\nabla^2\rho_A + \mathrm{v}\,\nabla.\rho_A + \frac{\partial\rho_A}{\partial t} = 0$$

For molar concentration:

$$-D_{AB}\nabla^2 c_A + \mathrm{V}\,\nabla.c_A + \frac{\partial c_A}{\partial t} = 0$$

3. If there is no fluid motion, no consumption or generation term, and constant density and diffusivity

For mass concentration:

$$\therefore\ \frac{\partial\rho_A}{\partial t} = D_{AB}\nabla^2\rho_A$$

For molar concentration:

$$\frac{\partial c_A}{\partial t} = D_{AB}\nabla^2 c_A \qquad (7)$$

Equation 7 referred to as <u>Fick's second law of diffusion</u>

Fick's second ''law'' of diffusion written in rectangular coordinates is

$$\frac{\partial c_A}{\partial t} = D_{AB}\left[\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} + \frac{\partial^2 c_A}{\partial z^2}\right]$$

in cylindrical coordinates is

$$\frac{\partial c_A}{\partial t} = D_{AB}\left[\frac{\partial^2 c_A}{\partial r^2} + \frac{1}{r}\frac{\partial c_A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 c_A}{\partial \theta^2} + \frac{\partial^2 c_A}{\partial z^2}\right]$$

and in spherical coordinates is

$$\frac{\partial c_A}{\partial t} = D_{AB}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c_A}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial c_A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 c_A}{\partial \phi^2}\right]$$

<u>Note:</u> what are the conditions at which there is no fluid motion? (bulk motion = 0.0)

The assumption of no fluid motion (bulk motion) restricts its applicability to:

a) Diffusion in solid
b) Stationary (stagnant) liquid
c) Equimolar counterdiffusion (for binary system of gases or liquids where N_A is equal in magnitude but acting in the opposite direction to N_B

$$N_A = -cD_{AB}\frac{dy_A}{dz} + y_A(N_A + N_B)$$

If $N_A = -N_B$ the bulk motion term will be cancelled from the above equation

4. If there is no fluid motion, no consumption or generation term, constant density and diffusivity and steady state conditions

For mass concentration:

$$\nabla^2 \rho_A = 0$$

For mass concentration:

$$\nabla^2 c_A = 0$$

Note: see page 438 in the reference book for the differential equation of mass transfer in different coordinate systems.

The general differential equation for mass transfer of component A, or the equation of continuity of A, written in rectangular coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

in cylindrical coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{A,r}) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

and in spherical coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\partial \theta}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}\right] = R_A$$

- **<u>Initial and Boundary conditions</u>**

To describe a mass transfer process by the differential equations of mass transfer the initial and boundary conditions must be specified.

Initial and boundary conditions are used to determine integration constants associated with the mathematical solution of the differential equations for mass transfer

1. <u>Initial conditions:</u>

It means the concentration of the diffusing species at the start (t = 0) expressed in mass or molar units.

$$at\ t = 0\ c_A = c_{A_o}\ _{(molar\ units)}$$

$$at\ t = 0\ \rho_A = \rho_{A_o}\ _{(mass\ units)}$$

where c_{A_o} and ρ_{A_o} are constant (defined values)

Note: Initial conditions are associated only with unsteady-state or pseudo-steady-state processes.

2. Boundary conditions:

It means the concentration is specified (known) at a certain value of coordinate (x, y or z).

Concentration is expressed in terms of different units, for example, molar concentration c_A, mass concentration ρ_A, gas mole fraction y_A, liquid mole fraction x_A, etc.

Types of boundary conditions:

(1) The concentration of the transferring species A at a boundary surface is specified. (The concentration of the species is known at the interface)

Examples:

I. For a liquid mixture in contact with a pure solid A, (liquid-solid interface) the concentration of species A in the liquid at the surface is the solubility limit of A in the liquid, c_{A_s}

II. For a contacting gas and liquid, (gas-liquid) where transferring species A is present in both phases, there are two ways to specify the concentration at the gas–liquid interface.

(i) if both of the species in the liquid phase are volatile, then the boundary condition at the gas–liquid surface is defined for an ideal liquid mixture by Raoult's law

$$p_{A_s} = x_A p_A$$

and from Dalton's law

$$y_{A_s} = \frac{p_{A_s}}{p}$$

or the surface concentration by the ideal gas law

$$c_{A_s} = \frac{p_{A_s}}{RT}$$

(ii) for solutions where species A is only weakly soluble in the liquid, Henry's law may be used to relate the mole fraction of A in the liquid to the partial pressure of A in the gas

$$p_A = H \cdot x_A$$

H: Henry's constant

(iii) at gas solid interface

$$c_{A_{solid}} = S \cdot p_A$$

$c_{A_{solid}}$: is the molar concentration of A within the solid at the interface in units of kg mol/m^3 and p_A is the partial pressure of gas phase species A over the solid in units of Pa.

S: solubility constant (partition coefficient)

(2) A reacting surface boundary is specified

$$N_A \Big|_{z=0} = k_c c_{A_s}^n$$

where k_c is a surface reaction rate constant with units of m/s. n is the reaction order

Note: the reaction may be so rapid that $c_{A_s} = 0$ if species A is the limiting reagent in the chemical reaction.

(3) The flux of the transferring species is zero at an impermeable boundary

$$N_A \Big|_{z=0} = -D_{AB} \frac{\partial c_A}{\partial z} \Big|_{z=0} = 0$$

$$or \ \frac{\partial c_A}{\partial z} \Big|_{z=0} = 0$$

where the impermeable boundary or the centerline of symmetry is located at z = 0

(4) The convective mass flux at the boundary surface is specified

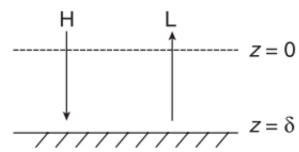
$$N_A \Big|_{z=0} = k \left(c_{A_s} - c_{A_\infty}\right)$$

Where k is the convection mass transfer coefficient

Solved problems:

Problem 1:

The following sketch illustrates the gas diffusion in the neighborhood of a catalytic surface. Hot gases of heavy hydrocarbons diffuse to the catalytic surface where they are cracked into lighter compounds by the reaction: H → 2L, the light products diffuse back into the gas stream.



a. Reduce the general differential equation for mass transfer to write the specific differential equation that will describe this steady-state transfer process if the catalyst is considered a flat surface. List all of the assumptions you have made in simplifying the general differential equation.
b. Determine the Fick's law relationship in terms of only compound H and insert it into the differential equation you obtained in part (a).
c. Repeat the solution for spherical catalyst surface.

Solution

a. The specific differential equation

Assumptions: steady state, unidirectional mass transfer.

$$\frac{\partial}{\partial x}N_{A,x} + \frac{\partial}{\partial y}N_{A,y} + \frac{\partial}{\partial z}N_{A,z} + \frac{\partial c_A}{\partial t} - R_A = 0$$

Apply these assumptions on the general equation we get the specific differential equation:

$$\frac{\partial}{\partial z}N_{H,z} + R_H = 0$$

b. Fick's law relationship in terms of only compound H

$$N_H = -cD_{HL}\frac{dy_H}{dz} + y_H(N_H + N_L)$$

but $N_L = -2N_H$

Fick's law in terms of H

$$N_H = -\frac{cD_{HL}}{(1+y_H)}\frac{dy_H}{dz}$$

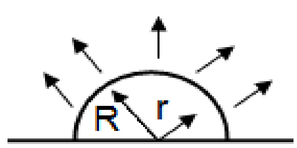
Problem 2:

A hemispherical droplet of liquid water, lying on a flat surface, evaporates by molecular diffusion through still air surrounding the droplet. The droplet initially has a radius R. As the liquid water slowly evaporates, the droplet shrinks slowly with time, but the flux of the water vapor is at a nominal steady state. The temperature of the droplet and the surrounding still air are kept constant. The air contains water vapor at an infinitely long distance from the droplet's surface.

a. After drawing a picture of the physical process, select a coordinate system that will best describe this diffusion process, list at least five reasonable assumptions for the mass-transfer aspects of the water-evaporation process and simplify the general differential equation for mass transfer in terms of the flux N_A.
b. What is the simplified differential form of Fick's equation for water vapor (species A)?

Solution:

a. The coordinate system that will describe



Basic assumptions:

1. Steady state conditions
2. No chemical reaction
3. Constant pressure and temperature
4. One dimensional mass transfer (r direction)
5. $N_{Air} = 0$

Apply these assumptions on the general differential equation:

$$\nabla . \vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

$$\therefore \nabla . \vec{N}_A = 0$$

For spherical coordinates and mass transfer in r-direction

$$\frac{1}{r^2} \frac{d}{dr} r^2 N_A = 0$$

b. The simplified differential form of Fick's equation for water vapor (species A)?

Assume water is A and air is B

$$N_A = -c D_{AB} \frac{d y_A}{dr} + y_A (N_A + N_B)$$

$$N_B = 0$$

$$N_A = -\frac{c D_{AB}}{(1 - y_A)} \frac{d y_A}{dr}$$

Problem 3:

A large deep lake, which initially had a uniform oxygen concentration of 1kg/m^3, has its surface concentration suddenly raised and maintained at 9 kg/m^3 concentration level. Reduce the general differential equation for mass transfer to write the specific differential equation for

a. the transfer of oxygen into the lake without the presence of a chemical reaction;
b. the transfer of oxygen into the lake that occurs with the simultaneous disappearance of oxygen by a first-order biological reaction.

Solution:

Assume oxygen = A and water = B

a. the transfer of oxygen into the lake without the presence of a chemical reaction

$$\nabla . \vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

Basic assumptions:

1. no chemical reaction occur
2. For deep lake (stationary liquid) we can assume v = 0
3. Unidirectional mass transfer (assume in z direction)

$$\frac{dN_A}{dz} + \frac{\partial c_A}{\partial t} = 0$$

But

$$N_A = -cD_{AB}\frac{dy_A}{dz} + c_A V$$

Since the liquid is stationary

$$\therefore N_A = -cD_{AB}\frac{dy_A}{dz}$$

$$\frac{dN_A}{dz} = -D_{AB}\frac{d^2 c_A}{dz^2}$$

$$\frac{\partial c_A}{\partial t} = D_{AB}\frac{d^2 c_A}{dz^2}$$

b. The transfer of oxygen into the lake that occurs with the simultaneous disappearance of oxygen by a first-order biological reaction.

Basic assumptions:

1. chemical reaction occur ($-R_A = k_r c_A$)
2. For deep lake (stationary liquid) we can assume v = 0
3. Unidirectional mass transfer (assume in z direction)

$$D_{AB}\frac{d^2 c_A}{dz^2} + \frac{\partial c_A}{\partial t} + k_r c_A = 0$$

where $k_r$is the reaction rate constant

Problem 4:

A liquid flows over a thin, flat sheet of a slightly soluble solid. Over the region in which diffusion is occurring, the liquid velocity may be assumed to be parallel to the plate and to be given by $V_x = ay$, where y is the vertical distance from the plate and a is a constant. Show that the equation governing the mass transfer, with certain simplifying assumptions, is

$$D_{AB}\left(\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2}\right) = ay\frac{\partial c_A}{\partial x}$$

List the simplifying assumptions, and propose reasonable boundary conditions.

Solution:

Basic assumptions:

1. Steady state conditions
2. Mass transfer in x and y directions
3. No chemical reaction occur
4. Constant temperature and pressure (constant concentration and diffusivity)
5. Velocity in y direction = 0

Apply these assumptions on the general differential equation:

$$\frac{\partial}{\partial x} N_{A,x} + \frac{\partial}{\partial y} N_{A,y} + \frac{\partial}{\partial z} N_{A,z} + \frac{\partial c_A}{\partial t} + R_A = 0$$

$$\boxed{\therefore \frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} = 0} \qquad \text{(i)}$$

$$N_{A,x} = -cD_{AB}\frac{dy_A}{dx} + c_A V_x$$

$$\therefore N_{A,x} = -D_{AB}\frac{dc_A}{dx} + ayc_A$$

$$N_{A,y} = -D_{AB}\frac{dc_A}{dy} + c_A V_y$$

But $V_y = 0$ (based on the assumptions)

$$\therefore N_{A,y} = -D_{AB}\frac{dc_A}{dy}$$

Substitute by the values of $N_{A,x}$ and $N_{A,y}$ in equation (i)

$$-D_{AB}\frac{\partial^2 c_A}{\partial x^2} + \text{ay}\frac{\partial c_A}{\partial x} - D_{AB}\frac{\partial^2 c_A}{\partial y^2} = 0$$

$$\therefore D_{AB}\left[\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2}\right] = \text{ay}\frac{\partial c_A}{\partial x}$$

Boundary conditions:

1. $c_A = c_{A_s}$ at $\text{y} = 0$
2. $c_A = 0$ at $\text{y} = \infty$
3. $c_A = 0$ at $\text{x} = 0$

Note:

The specific differential equation of mass transfer for a given system can be obtained by two methods:

1. Select the general equation according to the system coordinates and omit the unnecessary terms
2. Make a mass balance over a control volume, divide by the volume and take limits as values of length approaches zero

Supplementary data:

The general differential equation for mass transfer of component A, in rectangular coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

in cylindrical coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{A,r}) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

and in spherical coordinates is

$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\partial \theta}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}\right] = R_\mathrm{A}$$