Choose the best answer for each of the following questions:

The length of time, in minutes, for an airplane to obtain clearance for take off at a certain airport is a random variable $Y = 3X - 2$, where $X$ has the density function

$$f(x) = \begin{cases} 
\frac{1}{4}e^{-x/4}, & x > 0, \\
0, & \text{elsewhere}.
\end{cases}$$

Answer questions 1–4.

1. The mean of $X$ is \ldots
   (a) zero  \hspace{1cm} (b) $x/4$  \hspace{1cm} (c) 4  \hspace{1cm} (d) 1/4

2. The mean of $Y$ is \ldots
   (a) 10  \hspace{1cm} (b) -5/4  \hspace{1cm} (c) 4  \hspace{1cm} (d) -2

3. The second moment of $X$, i.e. $E[X^2] = \ldots$
   (a) 16  \hspace{1cm} (b) 64  \hspace{1cm} (c) 4  \hspace{1cm} (d) 32

4. The variance of $Y$ is \ldots
   (a) 48  \hspace{1cm} (b) 144  \hspace{1cm} (c) 46  \hspace{1cm} (d) 142

If a random variable $X$ is defined such that $E[(X - 1)^2] = 10$ and $E[(X - 2)^2] = 6$. Answer questions 5 and 6.

5. The mean of $X$ is \ldots
   (a) 16  \hspace{1cm} (b) $7/2$  \hspace{1cm} (c) $15/4$  \hspace{1cm} (d) $\sqrt{15}/2$

6. The standard deviation of $X$ is \ldots
   (a) $15/4$  \hspace{1cm} (b) 16  \hspace{1cm} (c) $\sqrt{15}/2$  \hspace{1cm} (d) $7/2$

7. A random variable is called a \ldots random variable if its set of possible outcomes is countable.
   (a) confounding  \hspace{1cm} (b) discrete  \hspace{1cm} (c) dependent  \hspace{1cm} (d) continuous

8. If $f(x)$ is a probability mass function of the discrete random variable $X$, then the sum of the probabilities, $\sum_x f(x)$ for each possible outcome $x$, must equal \ldots
   (a) -1  \hspace{1cm} (b) $1/2$  \hspace{1cm} (c) zero  \hspace{1cm} (d) 1
9. The length of time to play 18 holes of golf is an example of what type of random variables?
   (a) qualitative  (b) continuous  (c) discrete  (d) predictor

10. Determine the value $c$ so that the following function can serve as a probability distribution of the discrete random variable $X$:

\[ f(x) = c \left( \frac{2}{x} \right) \left( \frac{3}{3 - x} \right), \quad \text{for } x = 0, 1, 2. \]

(a) 10  (b) 3  (c) 1  (d) $1/10$

A random variable $X$ has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev’s theorem, find the answers of questions 7–10.

11. $P(|X - 10| \geq 3)$
   (a) at least $5/9$  (b) at most $4/9$  (c) 10  (d) at least $21/25$

12. $P(|X - 10| < 3)$
   (a) 10  (b) at least $21/25$  (c) at most $4/9$  (d) at least $5/9$

13. $P(5 < X < 15)$
   (a) at most $4/9$  (b) at least $5/9$  (c) at least $21/25$  (d) 10

14. The value of $c$ such that $P(|X - 10| \geq c) \leq 0.04$
   (a) 10  (b) at least $4/9$  (c) at least $5/9$  (d) 100

On a laboratory assignment, if the equipment is working, the density function of the observed outcome, $X$ is

\[ f(x) = \begin{cases} 
2(1 - x), & 0 < x < 1, \\
0, & \text{elsewhere.}
\end{cases} \]

Answer questions 15–17.

15. $P(X \leq 1/3)$
   (a) $3/16$  (b) $5/9$  (c) $1/18$  (d) $3/4$

16. The probability that $X$ will exceed 0.5 is …
   (a) $1/2$  (b) $5/9$  (c) $2/3$  (d) $1/4$

17. Given that $X$ will exceed 0.5, what is the probability that $X$ will be less than 0.75?
   (a) $3/4$  (b) $1/4$  (c) $3/16$  (d) $5/9$

The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable $X$ that has the following density function: (Answer questions 18–20)

\[ f(x) = \begin{cases} 
x, & 0 < x < 1, \\
2 - x, & 1 \leq x < 2, \\
0, & \text{elsewhere.}
\end{cases} \]

18. Find the probability that a family runs their vacuum cleaner less than 120 hours.
   (a) 0.18  (b) 0.5  (c) 0.68  (d) 0.375
19. Find the probability that a family runs their vacuum cleaner between 50 and 100 hours.
   (a) 0.375  (b) 0.18  (c) 1.44  (d) 0.68

20. Find the average number of hours per year that the families run their vacuum cleaners.
   (a) 0.75  (b) 0.375  (c) 0.5  (d) 1

Consider the random variable $X$ with the probability mass function $f(x)$, where
\[ f(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n \]

Answer questions 21–23.

21. The moment generating function of $X$ is \cdots
   (a) $n(pe^t + q)^{n-1}$  (b) $pe^t + q$  (c) $(pe^t + q)^n$  (d) $np$

22. The expected value of $X$ is \cdots
   (a) $np$  (b) $n(pe^t + q)^{n-1}$  (c) $npq$  (d) $pe^t + q$

23. If $Y = cX$, then the moment generating function of $Y$ is \cdots
   (a) $nc(pe^t + q)^{n-1}$  (b) $cpe^{ct} + q$  (c) $(pe^{ct} + q)^n$  (d) $cp$

Let $X$ be a random variable with the following probability distribution: (Answer questions 24–26)

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1/6</td>
<td>1/2</td>
<td>1/3</td>
</tr>
</tbody>
</table>

24. Find $F(6)$.
   (a) 2/6  (b) 4/6  (c) 1  (d) 1/2

25. Find $E[g(x)]$, where $g(x) = (2x + 1)^2$.
   (a) 186  (b) 209  (c) 46.5  (d) 5.5

26. Find the standard deviation of $X$.
   (a) 5.5  (b) 46.5  (c) 16.25  (d) 4.03

Determine whether each of the following statement is true or false. Answer questions 27–30

27. The expected value of a constant, $\alpha$ is zero.
   (a) True  (b) False

28. The variance of a random variable $X$ is the difference between the second moment and the first moment.
   (a) True  (b) False

29. For the random variable $W$, $f(w) = F(w) - F(w + 1)$
   (a) True  (b) False

30. Let $X$ be a random variable and $a, b$ are two constants, then $\sigma_{aX+b}^2 = a\sigma_X^2$
   (a) True  (b) False
Answers:

1. C
2. A
3. D
4. B
5. B
6. C
7. B
8. D
9. B
10. D
11. B
12. D
13. C
14. A
15. B
16. D
17. A
18. C
19. A
20. D
21. C
22. A
23. C
24. B
25. B
26. D
27. B
28. B
29. B
30. B