Complex Numbers Review

1. $(j \triangleq \sqrt{-1})$ is an imaginary number; it is the base of complex numbers.
2. a complex number has a *real* part and an *imaginary* part as: $(x + j\,y)$

1. Addition

$$(a + j\,b) \pm (c + j\,d) = (a + b) \pm j\,(c + d)$$

Examples:

1. $(2 + j\,3) + (4 + j\,5) = (5 + j\,8)$
2. $(2 + j\,3) - (4 + j\,5) = (-2 - j\,2)$
3. $\left(2\pi + j\,\sqrt{3}\right) - \left(-4 + j\,5\sin\left(\frac{\pi}{4}\right)\right) = (2\pi + 4) + j\left(\sqrt{3} - 5\sin\left(\frac{\pi}{4}\right)\right) \cong 10.28 - j\,1.80$

2. Multiplication

$$(a + j\,b) \times (c + j\,d) = (ac - bd) + j\,(bc + ad)$$

note: $j^2 = -1$

Examples:

1. $(2 + j\,3) \times (4 + j\,5) = (8 - 15) + j(12 + 10) = -7 + j\,22$
2. $\left(2\pi + j\,\sqrt{3}\right) \times \left(-4 + j\,5\sin\left(\frac{\pi}{4}\right)\right) \cong (6.28 + j\,1.73) \times (-4 + j\,3.54) = -31.24 + j\,15.31$

3. Polar Format

$$(x + j\,y) \longrightarrow \begin{cases} \sqrt{x^2 + y^2}\,\angle^{\tan^{-1}\left(\frac{y}{x}\right)} & x \geq 0 \\ \sqrt{x^2 + y^2}\,\angle^{180 + \tan^{-1}\left(\frac{y}{x}\right)} & x < 0 \end{cases}$$

$$a\,\angle\theta \longrightarrow a\cos\theta + j\,a\sin\theta$$

Examples:

1. $(2 + j\,3) \longrightarrow \sqrt{2^2 + 3^2}\,\angle\tan^{-1}\frac{3}{2} = 3.61\,\angle 56.31°$
2. $5\angle 30° \longrightarrow 5\cos 30 + j\,5\sin 30 = 4.33 + j\,2.5$

4. Division

$$\frac{(a + j\,b)}{(c + j\,d)} = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}\,\angle\left(\tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)\right)$$

5. Exercises

$$
\begin{aligned}
a &= 3 + j\,5 \\
b &= 8 - j\,7 \\
c &= -2 + j\,3 \\
d &= -10 - j\,4 \\
e &= 15\angle 30^\circ
\end{aligned}
$$

Calculate the followings:

1. $a + b$
2. $c \times d$
3. $(a + c) \div (b - d)$
4. polar forms of *a, b, c,* and *d*
5. Cartesian form of *e*
6. $(e - c) \times \frac{a}{d}$

Answers:

1. 11 –*j* 2
2. 32 –*j* 22
3. -0.018 +*j* 0.44
4. $a = 5.83\angle 59.04^\circ,\ b = 10.63\angle -41.19^\circ,\ c = 3.61\angle 123.7^\circ,\ d = 10.77\angle -158.2^\circ$
5. 12.99 +*j* 7.5
6. -4.99 –*j* 6.85