

MATH 110

قسم الرياضيات - كلية العلوم



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Chapter 1

Basics of Sets

- ✓ A set is a collection of objects. These objects are called members, or elements.
- ✓ Sets are denoted by uppercase letters such as X, Y, A, B , and their elements by lowercase letters such as x, y, a, b . The object x is a member of a set X is symbolized by $x \in X$. If x is not a member of a set X , then we denote that by $x \notin X$.

Example 1

The set of days of the week can be represented by listing

- ✓ For two sets X and Y , we define:
 - $X \setminus Y = \{x: x \in X \text{ and } x \notin Y\}$, called the complement of Y with respect to X .
 - $X \cap Y = \{x: x \in X \text{ and } x \in Y\}$, called the intersection of X and Y .
 - $X \cup Y = \{x: x \in X \text{ or } x \in Y\}$, called the union of X and Y .

✓ **Main sets of numbers**

The Natural numbers $\mathbb{N} = \{1, 2, 3, 4, \dots\}$.

The Integers numbers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$.

The Rational numbers $\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$.

✓ **For $a, b \in \mathbb{R}$ such that $a < b$:**

1. $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
2. $(a, \infty) = \{x \in \mathbb{R} : a < x\}$
3. $(-\infty, a) = \{x \in \mathbb{R} : x < a\}$
4. $(-\infty, \infty) = \{x \in \mathbb{R} : -\infty < x < \infty\} = \mathbb{R}$
5. $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
6. $[a, \infty) = \{x \in \mathbb{R} : a \leq x\}$
7. $(-\infty, a] = \{x \in \mathbb{R} : x \leq a\}$
8. $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$
9. $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$.

Example

a) $(2, 6] \setminus (4, 8) = (2, 4]$.

b) $(1, 3] \cup (2, 7) = (1, 7)$.

c) $(0, 6] \cap (2, 8) = (2, 6]$.

Equations and Inequalities

✓ To solve a linear equation we follow these steps:

- Eliminate each bracket.
- Put the terms of variable on one side and the constants on the other side.
- Divide both sides by the coefficient of the variable.

Example

Solve the equation $4x - 1 = 2 \left(3 - \frac{x}{2} \right)$.

Solution

$$4x - 1 = 2 \left(3 - \frac{x}{2} \right) \Rightarrow 4x - 4 = 6 - x$$

$$\Rightarrow 4x + x = 6 + 4 \Rightarrow 5x = 10 \Rightarrow x = \frac{10}{5} \Rightarrow x = 2.$$

✓ For $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$ and $a \neq 0$. The following formula will solve any equation of this kind

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To apply this formula we have to determine a, b , and c correctly. The expression $b^2 - 4ac$, called the distinguisher, will distinguish the roots as follows:

- If $b^2 - 4ac < 0$, then there is no real root.
- If $b^2 - 4ac = 0$, then there is only one repeat root.
- If $b^2 - 4ac > 0$, then there are two distinct roots.

Example 2

Solve the equation $x^2 - 3x - 10 = 0$.

Solution

$$\because a = 1, b = -3, c = -10$$

$$\therefore x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 40}}{2} = \frac{3 \pm 7}{2}$$

Thus the first root is $x_1 = \frac{3+7}{2} = 5$, and the second root is $x_2 = \frac{3-7}{2} = -2$.

Example

Solve the equation $x^2 - 3x = 0$.

Solution

$$x^2 - 3x = 0 \Rightarrow x(x - 3) = 0 \Rightarrow \text{either } x = 0 \text{ or } x = 3.$$

Example

Solve the equation $x^2 - 9 = 0$.

Solution

$$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3.$$

Example

Solve the equation $x^2 - 3x - 10 = 0$.

Solution

$$x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0 \Rightarrow$$

either $x - 5 = 0 \Rightarrow x = 5$ **or** $x + 2 = 0 \Rightarrow x = -2$.

Inequalities.

✓ **For** $a, b, c \in \mathbb{R}$:

- $a < b \Rightarrow a + c < b + c.$
- $a < b$ and $c > 0 \Rightarrow ac < bc.$
- $a < b$ and $c < 0 \Rightarrow ac > bc.$
- $0 < a < b \Rightarrow \frac{1}{a} > \frac{1}{b}.$

All of the properties above are true if we replace $<$ by \leq .

Example

Solve the inequality $x + 6 < 4(x - 1).$

Solution

$$x + 5 < 4(x - 1) \Rightarrow x + 5 < 4x - 4 \Rightarrow x - 4x < -4 - 5 \Rightarrow -3x < -9 \Rightarrow x > \frac{-9}{-3} \Rightarrow x > 3.$$

So, the solution set is

$$\{x \in \mathbb{R}: x \geq 3\} = [3, \infty).$$

Absolute value

For any $x \in \mathbb{R}$, we define the absolute value of x , denoted by $|x|$, to be the distance on the real line between x and 0. Thus

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

The absolute value has the following properties:

For any $x, a \in \mathbb{R}$, with $a > 0$:

- $|x| = a \Leftrightarrow x = a \text{ or } x = -a.$
- $|x| < a \Leftrightarrow -a < x < a.$
- $|x| > a \Leftrightarrow x > a \text{ or } x < -a.$

The last two properties are true if we replace $<$ by \leq .

Example

Solve the inequality $|x - 2| \leq 4$.

Solution

$|x - 2| \leq 4 \Rightarrow -4 \leq x - 2 \leq 4 \Rightarrow -2 \leq x \leq 6$. The solution set is $[-2, 6]$.

- ✓ The distance between two real numbers x and y on the real line we can be measured by $|x - y|$.
- ✓ The distance between two points (x_1, y_1) and (x_2, y_2) in the plane can be measured by the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Example

Find the distance between $(3, 5)$ and $(2, 3)$.

Solution

$$\sqrt{(3 - 2)^2 + (5 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$$

Lines

- ✓ The slope of a line passing through the points (x_1, y_1) and (x_2, y_2) , denoted by m , is given by

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- ✓ An equation of a line which has slope m and passing through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1).$$

Example

Find the slope of the line passing through the points (3 , 1) and (1 , 5).

Solution

The slope $m = \frac{1-5}{3-1} = \frac{-4}{2} = -2$.

Example

Find an equation of the line has slope -2 and passing through the point (2 , 1).

Solution

An equation is $y - 1 = 2(x - 2) \Rightarrow y - 1 = 2x - 4 \Rightarrow y - 2x + 3 = 0$.

Example

Find an equation of the line passing through the points (3 , 1) and (1 , 5).

Solution

First, we have to find the slope. $m = \frac{1-5}{3-1} = -2$. **Then we choose any one of the two points. Let us take the first point, so, an equation is** $y - 1 = -2(x - 3) \Rightarrow y - 1 = -2x + 6 \Rightarrow y + 2x - 7 = 0$.