4.1 Preliminary

(1) Use $y(x) = c_1\cos(5x) + c_2\sin(5x)$ to determine that

$$y'' + 25y = 0$$

satisfies the boundary conditions $y(0) = 6$, $y'(\pi) = -9$.

(2) Determine whether the given set of functions is linearly independent or dependent on the given interval.

(a) $f_1(x) = x^2$, $f_2(x) = 1-x^2$, $f_3(x) = 2+x^2$ on $(-\infty, \infty)$.

(b) $f_1(x) = \cos(x + \frac{\pi}{2})$, $f_2(x) = \sin(x)$ on $(-\infty, \infty)$.

(3) Verify that the given function form a fundamental set of solutions of the differential equation on the indicated interval

(a) $2t^2y'' + ty' - 3y = 0$, $y_1(t) = t^{-1}, y_2(t) = t^{\frac{3}{2}}$ on $(0, \infty)$

(b) $y^(4) + y'' = 0; 1, x, \cos(x), \sin(x)$ on $(-\infty, \infty)$

(4) Verify that the given two parameter family of functions is the general solution of the non homogeneous differential equation on the indicated interval

(a) $y'' - 2y' + y = e^x$
$y = c_1e^x + c_2xe^x + \frac{1}{2}x^2e^x on(\infty, \infty)$

4.3 Homogenous linear equation with constant coefficients

Find the general solution of the given differential equations:

(1) $2y'' + 2y' + y = 0$

(2) $y'' + 2y' + y = 0$

(3) $y'' - 2y' + 5y = 0$

(4) $y'' + 9y = 0$

(5) $y'' + 6y' + 13y = 0$

(6) $2y''' - y'' + 36y' - 18y = 0$

(7) $2y^{(4)} + 11y''' - 4y'' - 69y' + 34y = 0$

(8) $(D^5 + D^4 - 7D^3 - 11D^2 - 8D - 12)y = 0$

Solve the IVP to the given DEs.

(1) $(D^3 + 7D^2 + 19D + 13)y = 0,\ y(0) = 0,\ y'(0) = 2,\ y''(0) = -12$

(2) $y'' + 2y' + 4y = 0, y(0) = 1, y'(0) = -1$

4.4 Undetermined coefficients

Solve the given DEs. by undetermined coefficients

(1) $y'' - 3y' + 2y = 2x^3 - 9x^2 + 6x$

(2) $y'' + 6y' + 13y = 60x\cos x + 26$

(3) $y'' - y' - 2y = 6x + 6e^{-x}$

(4) $y'' - y = e^x - 4$

(5) $y'' + 4y' + 3y = 15e^{2x} + e^{-x}$

(6) $y''' - 3y' - 2y = 100\sin 2x$

(7) $y'' - y = e^{-x}(2\sin x + 4\cos x)$

Solve the IVP to the given DEs. by undetermined coefficients.

(1) $y'' + y = 10e^{2x}$ $y(0) = y'(0) = 0$

(2) $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 5x = 10,\ x(0) = x'(0) = 0$

(3) $y'' - 4y = 2 - 8x,\ y(0) = 0, y'(0) = 5$

Solve the given boundary value problem
$y'' + 2y' + y = x,\ y(0) = -3, y(1) = -1$

4.5 Undetermined coefficients (annihilator approach)

Verify that the given differential operator annihilates the indicated function.

(1) $D^2 + 49$, $y = 2\cos(7x) - 5\sin(7x)$

(2) D^3, $y = 5x^2 - 1$

Find linearly independent functions that are annihilated by the given differential operator.

(1) D^4

(2) $D^2(D+1)(D^2+2D+1)$

(3) $(D-4)(2D+5)$

Find a linearly differential operator that annihilates the given function .

(1) $x^4 - 3x$

(2) $e^x \sin(3x) + \cos(4x)$

(3) $5\sin t + t\cos 3t + e^{-6t}$

Solve the following DE by using undetermined coefficients (annihilation approach).

(1) $y'' - y = x - 1$

(2) $y'' - 5y' + 6y = 2e^x$

(3) $y'' + 4y = \sin x$

(4) $y'' + 4y' + 4y = 18e^{-2x}\cos(3x)$

4.6 Variation of parameters

Solve each DE by variation of parameters.

(1) $y'' + y = \csc x \cot x$

(2) $y'' + y = \sec^2 x$

(3 $y'' - 3y' + 2y = \cos(e^{-x})$

(4) $y'' - y = \sinh(2x)$

(5) $y'' - 3y' + 2y = \dfrac{e^{2x}}{1 + e^{2x}}$

4.7 Cauchy-Euler equation

Find the general solution to the following DE.

(1) $x^2y'' - 7xy' + 16y = 0$

(2) $x^2y'' + 3xy' + 4y = 0$

(3) $x^2y'' + xy' - y = \ln x$

(4) $x^2y'' - 2y = x^3e^x$

Solve the given IVP.

$2x^2y'' + 3xy' - 15y = 0, \ y(1) = 0, y'(1) = 1$

4.8 Solving system of DEs. by elimination

Solve the given system of differential equations by systematic elimination.

(1)

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = x - y$$

(2)

$$\frac{dx}{dt} = tx + y\sin t$$
$$\frac{dy}{dt} = t^2x + ty + 1$$

Solve the given IVP:

$$\frac{dx}{dt} = y - 1$$
$$\frac{dy}{dt} = -3x + 2y, \quad x(0) = y(0) = 0$$