Chapter Seven

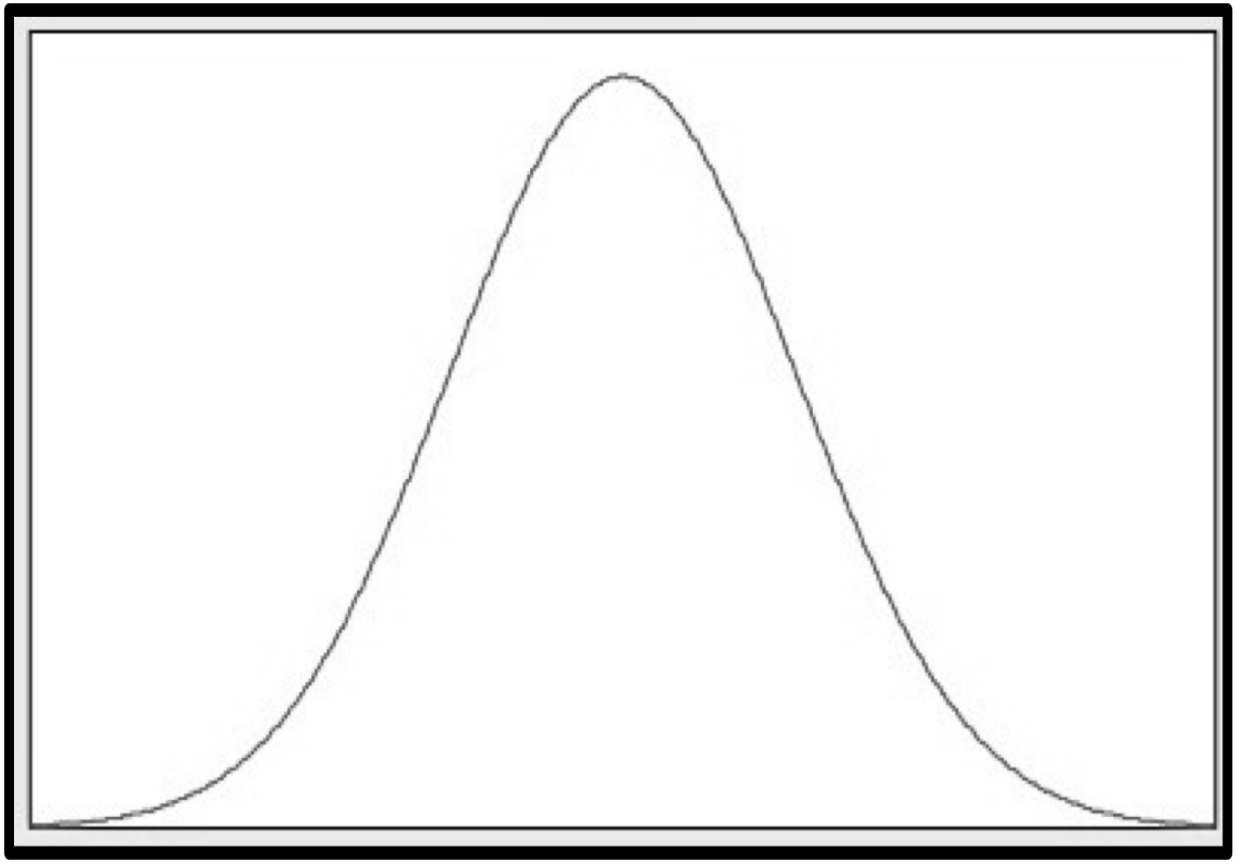
The Normal Distribution

7-1 Introduction

- Many continuous variables have distributions that are bell-shaped and are called **approximately normally distributed variables**, such as the heights of adult men, cholesterol level of adults.

- A normal distribution is also known as the **bell curve** or the **Gaussian distribution**.

7-1 Introduction



7-2 Normal and Skewed Distributions

- A **normal distribution** is a continuous, symmetric, bell-shaped distribution of a variable.

- If the data values are evenly distributed about the mean, the distribution is said to be **symmetrical**. (***mean=median=mode***)

- If the majority of the data values fall to the left or right of the mean, the distribution is said to be **skewed**.

7-3 Normal Distribution Properties

- The shape and position of the normal distribution curve depend on two parameters, the **mean** and the **standard deviation**.
- The mean, median, and mode are equal and located at the center of the distribution.
- The normal distribution curve is **unimodal** (i.e., it has only one mode).
- The curve of the normal distribution is continuous, i.e., there are no gaps. For each value of ***x***, there is a corresponding value of ***y***.
- The total area under the normal distribution curve is equal to 1.00 or 100%.

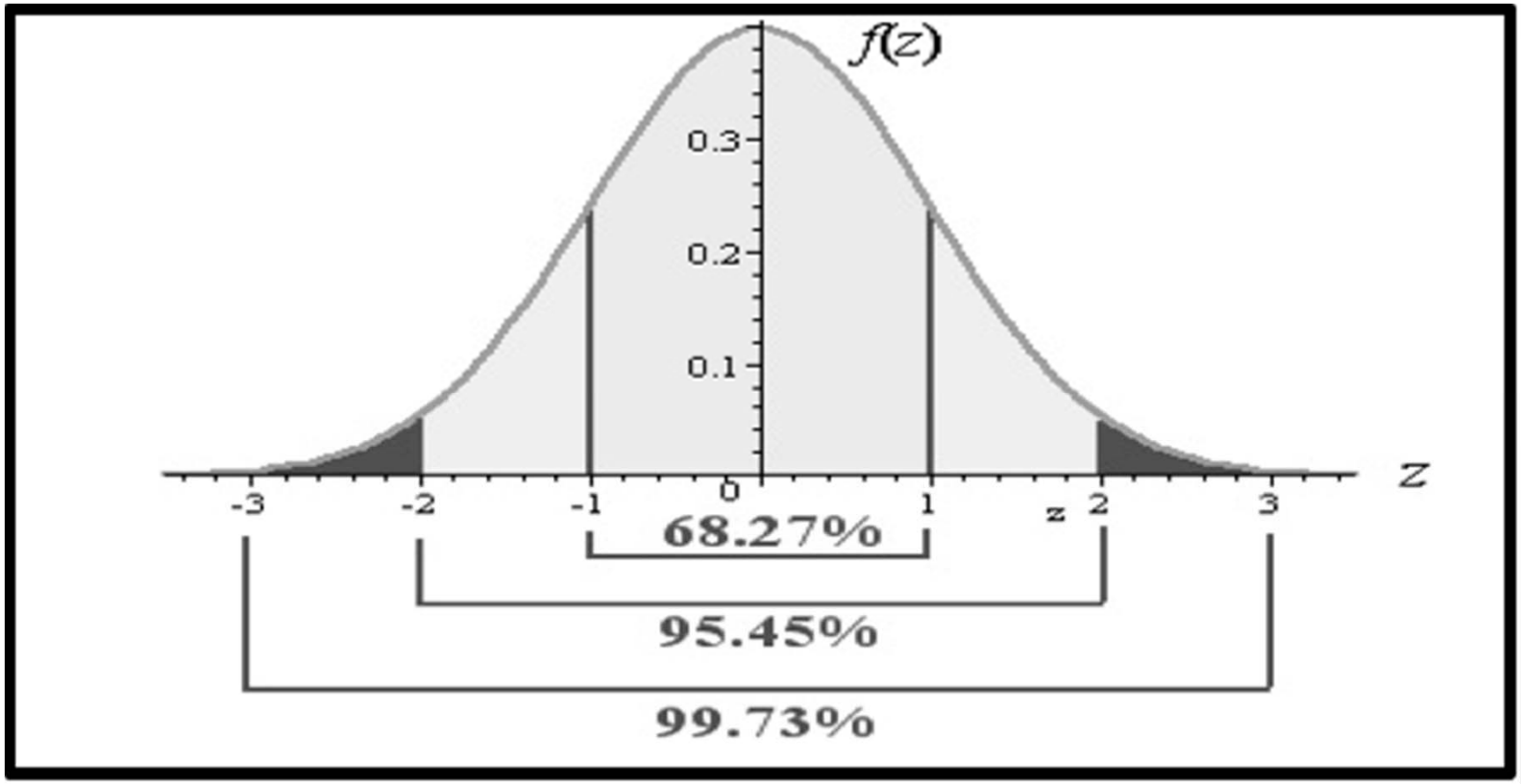
7-3 Normal Distribution Properties

- The area under the normal curve that lies within
  - one standard deviation of the mean is approximately 0.68 (68%).
  - two standard deviations of the mean is approximately 0.95 (95%).
  - three standard deviations of the mean is approximately 0.997 (99.7%).

Area Under Curve	Normal Distribution	Standard Normal Distribution
0.68	$(\mu-\sigma, \mu+\sigma)$	$(-1,+1)$
0.95	$(\mu-2\sigma, \mu+2\sigma)$	$(-2,+2)$
0.997	$(\mu-3\sigma, \mu+3\sigma)$	$(-3,+3)$

7-3 Normal Distribution Properties

- Areas Under the Normal Curve



7-4 Standard Normal Distribution

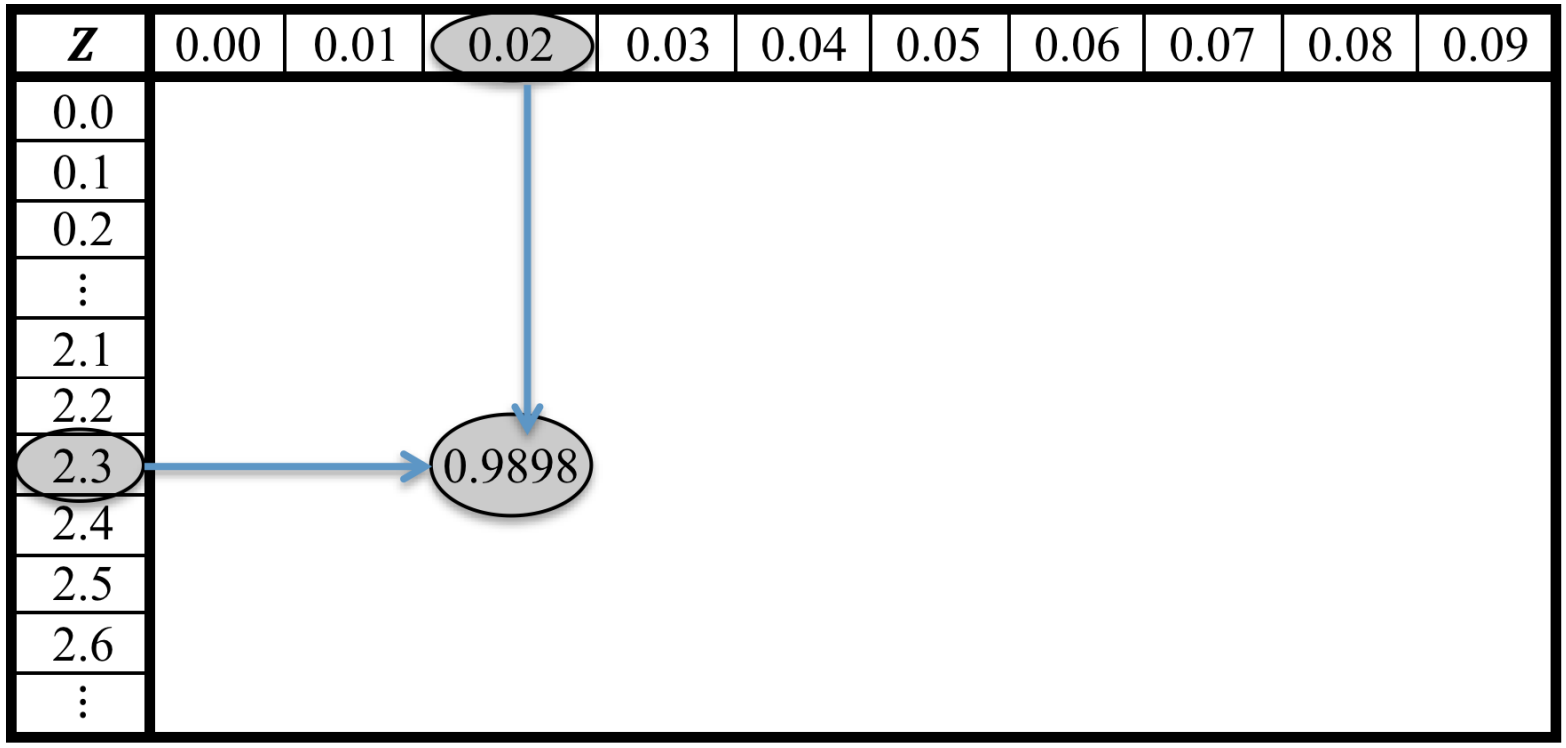
- The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

- All normally distributed variables can be transformed into the standard normally distributed variable by using the ***Z*** value which is the number of standard deviations that a particular *X* value is away from the mean:

$$Z = \frac{x - \mu}{\sigma}$$

7-5 Area Under the Standard Normal distribution Curve

- The table of the standard normal distribution gives the probability to the left of the values, ***P(Z < a)***
- Example: ***P(Z < 2.32) = 0.9898***



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0										
0.1										
0.2										
⋮										
2.1										
2.2										
2.3					0.9898					
2.4										
2.5										
2.6										
⋮										

7-5 Area Under the Standard Normal distribution Curve

- Note the following
  - $P(Z < a) = P(Z \leq a)$
  - $P(a < Z < b) = P(a \leq Z \leq b) = P(Z < b) - P(Z < a)$
  - $P(Z > a) = P(Z \geq a) = 1 - P(Z < a)$

7-5 Area Under the Standard Normal distribution Curve

- Example:
  - If the income of 10000 families fallows a normal distribution with mean 1800 SR. and standard deviation 300 SR. find the following:
    - The probability of a family income is less than 2550 SR.

$$P(x < 2550) = P\left(Z < \frac{2550 - 1800}{300}\right) = P(Z < 2.50) = 0.9938$$

    - The probability of a family income is less than 1300 SR.

$$P(x < 1300) = P\left(Z < \frac{1300 - 1800}{300}\right) = P(Z < -1.67) = 0.0475$$

    - The probability of a family income is greater than 2400 SR.

$$P(x > 2400) = P\left(Z > \frac{2400 - 1800}{300}\right) = P(Z > 2.00) = 1 - P(Z < 2.00)$$
$$= 1 - 0.9772 = 0.0228$$

7-5 Area Under the Standard Normal distribution Curve

- Example:
  - If the income of 10000 families fallows a normal distribution with mean 1800 SR. and standard deviation 300 SR. find the following:
    - The probability of a family income is greater than 1500 SR.

$$P(x > 1500) = P\left(Z > \frac{1500 - 1800}{300}\right) = P(Z > -1.00) = 1 - P(Z < -1.00)$$
$$= 1 - 0.1587 = 0.8413$$

    - The probability of a family income is between 1650 SR. and 2250 SR.

$$P(1650 < x < 2250) = P\left(\frac{1650 - 1800}{300} < Z < \frac{2250 - 1800}{300}\right)$$
$$= P(-0.50 < Z < 1.50) = P(Z < 1.50) - P(Z < -0.50)$$
$$= 0.9332 - 0.3085 = 0.6247$$

    - The number of families that have income greater than 1500 SR.

$$(10000)P(x > 1500) = (10000)(0.8413) = 8413 \text{ families}$$

7-5 Area Under the Standard Normal distribution Curve

- Example:
  - The lifetime of one type of microwaves follows a normal distribution with mean 3 years and standard deviation 1 year. If one microwave was chosen randomly.
    - What is the probability that its lifetime will be greater than 2 years?

$$P(x > 2) = P\left(Z > \frac{2-3}{1}\right) = P(Z > -1.00) = 1 - P(Z < -1.00) = 1 - 0.1587 = 0.8413$$

    - If the microwaves have warranty for one year, what is the percentage of microwaves that the factory has to exchange with free new ones?

$$(100)P(x < 1) = (100)P\left(Z < \frac{1-3}{1}\right) = (100)P(Z < -2.00) = (100)(0.0228) = 2.28\%$$

7-6 Distribution of Sample Means

- A **sampling distribution of sample means** is a distribution obtained by using the means computed from random samples of a specific size taken from a population.

- **Sampling error** is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

7-6 Distribution of Sample Means

- The Central Limit Theorem
  - As the sample size ***n*** increases, the shape of the distribution of the sample means taken with replacement from a population with mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$ will approach a normal distribution.
  - Thus, the mean of the sample means equals the population mean, $\boldsymbol{\mu_{\bar{x}} = \mu}$, and the standard deviation of the sample means which is called the **standard error of the mean** is $\boldsymbol{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$
  - The central limit theorem can be used to answer questions about sample means in the same manner that the normal distribution can be used to answer questions about individual values.
  - A new formula must be used for the *z* values: $\boldsymbol{Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$

7-6 Distribution of Sample Means

- Example:
  - A.C. Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of TV per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 32 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch TV is greater than 26.3 hours.

$$P(\bar{x} > 26.3) = P\left(Z > \frac{26.3 - 25}{\frac{3}{\sqrt{32}}}\right) = P(Z > 2.45)$$

$$= 1 - P(Z < 2.45) = 1 - 0.9929 = 0.0071$$

7-6 Distribution of Sample Means

- Example:
  - The average age of a vehicle registered in the United States is 8 years, or 96 months. Assume the standard deviation is 16 months. If a random sample of 36 cars is selected, find the probability that the mean of their age is between 90 and 100 months.

$$P(90 < \bar{x} < 100) = P\left(\frac{90 - 96}{16/\sqrt{36}} < Z < \frac{100 - 96}{16/\sqrt{36}}\right)$$

$$= P(-2.25 < Z < 1.50) = P(Z < 1.50) - P(Z < -2.25)$$

$$= 0.9332 - 0.0122 = 0.9210$$

7-6 Distribution of Sample Means

- Example:
  - The average number of pounds of meat that a person consumes a year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.
    - Find the probability that a person selected at random consumes less than 224 pounds per year.

$$P(x < 224) = P\left(Z < \frac{224 - 218.4}{25}\right) = P(Z < 0.22) = 0.5871$$

    - If a sample of 40 individual is selected, find the probability that the mean of the sample will be less than 224 pounds per year.

$$P(\bar{x} < 224) = P\left(Z < \frac{224 - 218.4}{25/\sqrt{40}}\right) = P(Z < 1.42) = 0.9222$$

7-7 Standard Normal Distribution tables

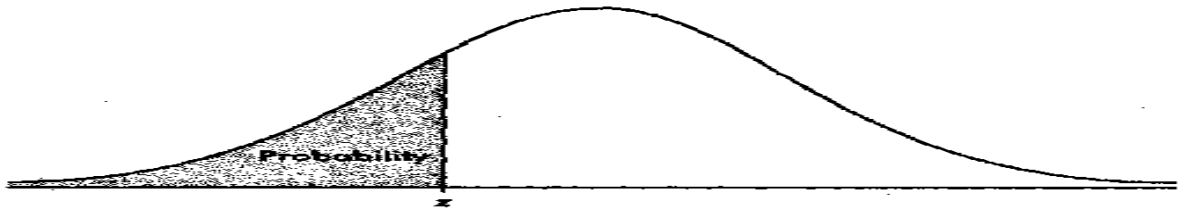


TABLE A: STANDARD NORMAL PROBABILITIES

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

7-7 Standard Normal Distribution tables

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998