Chapter Five

Probability

5-1 Introduction

- **Probability** as a general concept can be defined as the chance of an event occurring.

- Probability are used in games of chance, insurance, investments, and weather forecasting, and in various areas.

5-2 Basic Concepts

- A **probability experiment** is a chance process that leads to well-defined results called outcomes.
- An **outcome** is the result of a single trial of a probability experiment.
- A **sample space** is the set of all possible outcomes of a probability experiment.
- An **event** consists of a set of outcomes of a probability experiment.
- An event with one outcome is called a **simple event** and with more than one outcome is called **compound event**.

5-2 Basic Concepts

- Example:
  - Find the sample space for the gender of the children if a family has three children and give an example for simple event and another one for a compound event. Use ***B*** for boy and ***G*** for girl.

  There are two gender and three children, so there are *23=8*

  possibilities as shown here,

  BBB BBG BGB GBB GGG GGB GBG BGG

  So, the sample space is

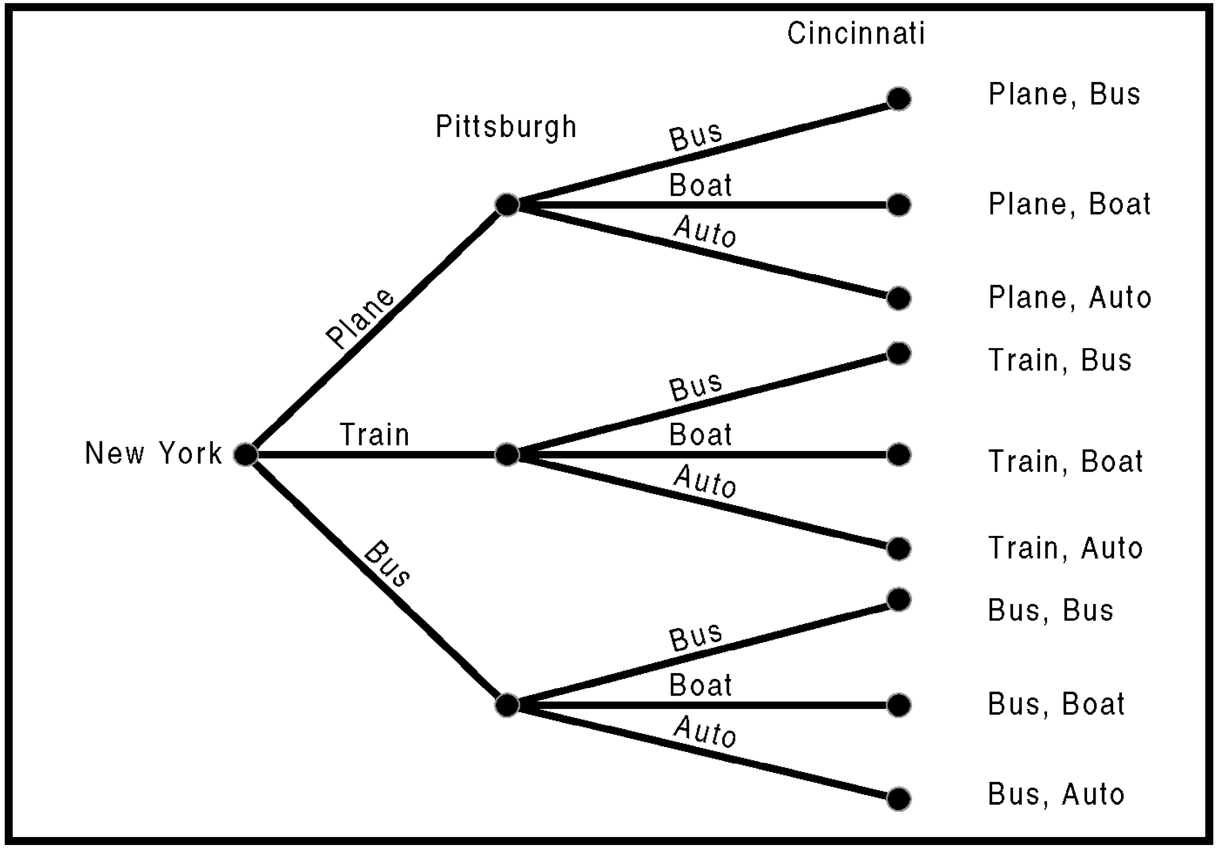
  S={ BBB,BBG,BGB,GBB,GGG,GGB,GBG,BGG }

  Simple event as ***E={BBB}***

  Compound event as ***E={BBG, BGB, GBB}***

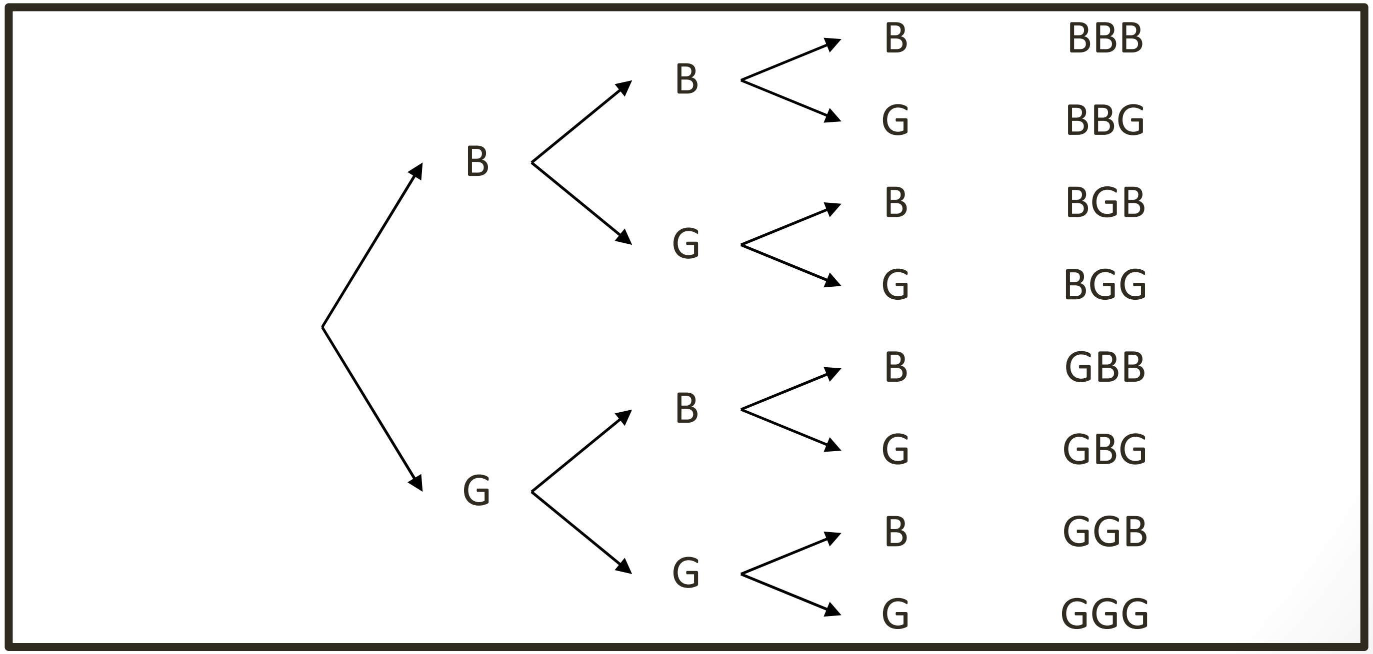
5-2 Basic Concepts

- A **tree diagram** is a device used to list all possibilities of a sequence of events in a systematic way.



5-2 Basic Concepts

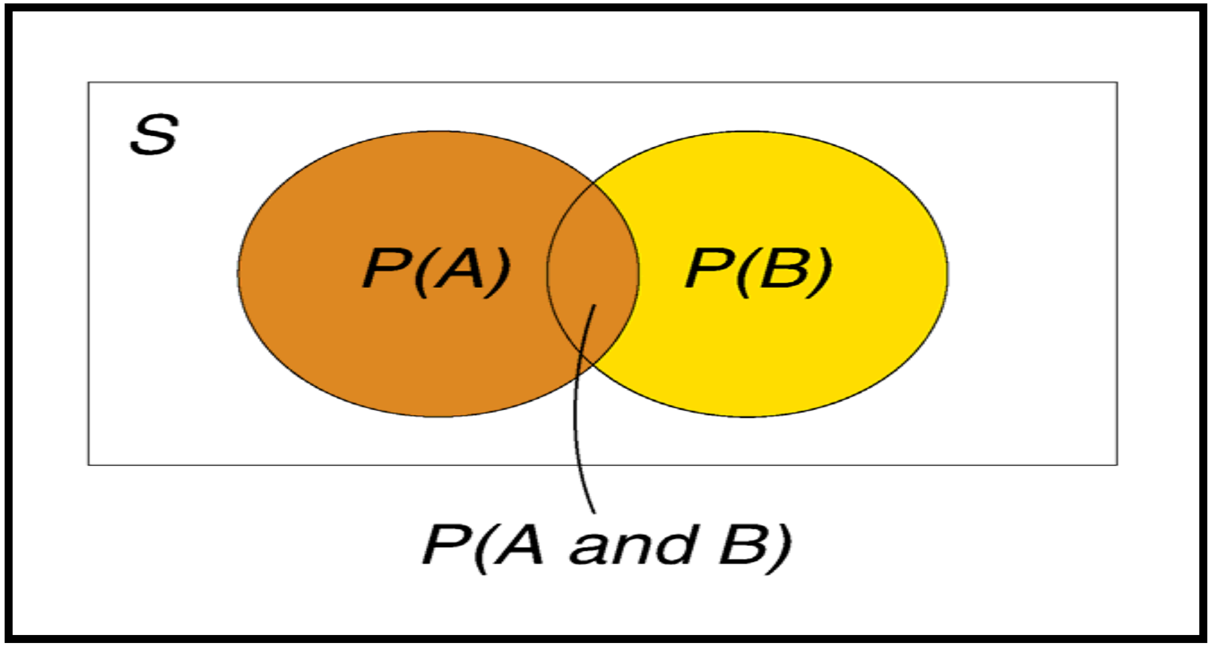
- Example:
  - Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl. Use a tree diagram to find the sample space for the gender of the three children.



5-2 Basic Concepts

- **<u>Equally likely events</u>** are events that have the same probability of occurring.

- **<u>Venn diagrams</u>** are used to represent probabilities pictorially.



5-3 Classical Probability

- **<u>Classical probability</u>** uses sample spaces to determine the numerical probability that an event will happen. It assumes that all outcomes in the sample space are equally likely to occur.

- The probability of an event E can be defined as

$$\boldsymbol{P(E) = \frac{n(E)}{n(S)} = \frac{Number\ of\ outcomes\ in\ E}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}}$$

5-3 Classical Probability

- Example:
  - If a family has three children, find the probability that two of the children are girls.

    The sample space is

    S={ BBB,BBG,BGB,GBB,GGG,GGB,GBG,BGG }

    n(S)=8

    The event of two girls is

    E={GGB, GBG, BGG}

    n(E)=3

    The probability that two of the children are girls is

$$\boldsymbol{P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}}$$

5-3 Classical Probability

Rounding Rule for Probabilities

- Note:
  - Probabilities should be expressed as reduced fractions or rounded to three decimal places.

  - When the probability of an event is an extremely small decimal, it should be rounded to the first nonzero digit after the decimal point. A number 0.00003467 can be rounded as 0.00003 or 0.000006589 can be rounded as 0.000007.

5-3 Classical Probability

Probability Rules

- The probability of an event ***E*** is a number (either a fraction or decimal) between and including **0** and **1**, $0 \leq P(E) \leq 1$.
- If an event ***E*** cannot occur (i.e., the event ***E*** contains no members in the sample space), the probability is zero.
- If an event ***E*** is certain, then the probability of *E* is one.
- The sum of the probabilities of the outcomes in the sample space is one.

5-3 Classical Probability

- Example:
  - When a single die is rolled, find the probability of getting a 9.

  Since the sample space is ***S={1,2,3,4,5, 6}***, it is impossible to get a 9,

$$P(9) = \frac{n(9)}{n(S)} = \frac{0}{6} = 0$$

- Example:
  - When a single die is rolled, what is the probability of getting a number less than 7?

  Since all outcomes in the sample space are less than 7,

$$P(number\ less\ than\ 7) = \frac{n(number\ less\ than\ 7)}{n(S)} = \frac{6}{6} = 1$$

5-4 Complementary Events

- The **complement of an event *E*** is the set of outcomes in the sample space that are not included in the outcomes of event ***E***. The complement of ***E*** is denoted by $\overline{\boldsymbol{E}}$

  - Rule for Complementary Events $\boldsymbol{P(E) + P(\overline{E}) = 1}$

  - $\boldsymbol{P(\overline{E}) = 1 - P(E)}$ or $\boldsymbol{P(E) = 1 - P(\overline{E})}$

  - Complementary events are mutually exclusive.

5-4 Complementary Events

- Example:
  - Find the complement of each event.
    - a- Rolling a die and getting a 4.

      Getting a 1,2,3,5 or 6

    - b- Selecting a month and getting a month that begins with a J.

      Getting February, March, April, May, August, September, October, November or December

    - c- Selecting a day of the week and getting a weekday.

      Getting Thursday or Friday

5-4 Complementary Events

- Example:
  - If the probability that a person lives in an industrialized country of the world is ***1/5***, find the probability that a person does not live in an industrialized country.

P(living in an industrialized country) = 1/5

P(not living in an industrialized country)

= 1 - P(living in an industrialized country)

= 1 - (1/5) = 4/5

5-5 Empirical Probability

- **Empirical probability** relies on actual experience to determine the likelihood of outcomes.

- Given a frequency distribution, the probability of an event being in a given class is:

$$P(E) = \frac{frequency\ of\ the\ class}{total\ frequecies\ in\ the\ distribution} = \frac{f}{n}$$

5-5 Empirical Probability

- Example:
  - The frequency distribution of blood types for sample of 50 people as follow:

Blood Type	Frequency
A	22
B	5
O	21
AB	2

find the following probabilities.

5-5 Empirical Probability

- A person has type O blood.

$$P(O) = \frac{f_O}{n} = \frac{21}{50}$$

- b- A person has type A or type B blood.

$$P(A \text{ or } B) = \frac{f_A}{n} + \frac{f_B}{n} = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$$

- c- A person has neither type A nor type O blood.

$$P(\text{neither } A \text{ nor } O) = P(B \text{ or } AB) = \frac{f_B}{n} + \frac{f_{AB}}{n} = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$$

Or

$$P(\text{neither } A \text{ nor } O) = 1 - P(A \text{ or } O) = 1 - \left(\frac{f_A}{n} + \frac{f_O}{n}\right)$$

$$= 1 - \left(\frac{22}{50} + \frac{21}{50}\right) = 1 - \frac{43}{50} = \frac{7}{50}$$

- d- A person does not have type AB blood.

$$P(\text{not } AB) = 1 - P(AB) = 1 - \frac{f_{AB}}{n} = 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$$

5-6 Mutually Exclusive Events

- Two events are **mutually exclusive** if they cannot occur at the same time (they have no intersection, which means there are no outcomes in common).

- Two events are **not mutually exclusive** if they can occur at the same time (they have intersection, which means there are outcomes in common).

- The probability of two or more events can be determined by the **addition rules**.

5-6 Mutually Exclusive Events

- Example:
  - Determine which events are mutually exclusive and which are not, when a single die is rolled.
    - a- Getting an odd number and getting an even number.

      The events are mutually exclusive; since the first event can be 1, 3 or 5 and the second event can be 2, 4 or 6.
    - b- Getting a 3 and getting an odd number.

      The events are not mutually exclusive, since the first event is a 3 and then second event can be 1, 3 or 5. Hence, 3 is contained in both events.

5-6 Mutually Exclusive Events

- c- Getting an odd number and getting a number less than 4.

  The events are not mutually exclusive, since the first event can be 1, 3 or 5 and the second event can be 1, 2 or 3. Hence, 1 and 3 are contained in both events.

- d- Getting a number greater than 4 and getting a number less than 4.

  The events are mutually exclusive, since the first event can be 5 or 6 and the second event can be 1, 2 or 3.

5-6 Mutually Exclusive Events

- When two events ***A*** and ***B*** are mutually exclusive, the probability that ***A*** or ***B*** will occur is:

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$$

- When two events ***A*** and ***B*** are not mutually exclusive, the probability that ***A*** or ***B*** will occur is:

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

5-6 Mutually Exclusive Events

- Example:
  - A box contains 3 glazed doughnuts, 4 jelly doughnuts and 5 chocolate doughnuts. If a person selects a doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut.

  The total number of doughnuts in the box is 12 and the event are mutually exclusive, so

$$\boldsymbol{P(Glazed\ or\ Chocolate) = P(Galzed) + P(Chocolate)}$$

$$\boldsymbol{= \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}}$$

5-6 Mutually Exclusive Events

- Example:
  - A day of the week is selected at random. Find the probability that it is a weekend day (Thursday or Friday)

    The total number of days in week is 7 and the event are mutually exclusive, so

$$\boldsymbol{P(Thursday\ or\ Friday) = P(Thursday) + P(Friday)}$$

$$\boldsymbol{= \frac{1}{7} + \frac{1}{7} = \frac{2}{7}}$$

5-6 Mutually Exclusive Events

- Example:
  - In a hospital unit there are 8 nurses and 5 physicians shown in the following table

Staff	Female	Male	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

If a staff is selected, find the probability that the subject is a nurse or a male.

The events are not mutually exclusive and the sample space is

$$\boldsymbol{P(Nurse\ or\ Male) = P(Nurse) + P(Male)}$$
$$\boldsymbol{-P(Nurse\ and\ Male) = \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \frac{10}{13}}$$

5-7 Independent Events

- Two events ***A*** and ***B*** are **<u>independent</u>** if the fact that ***A*** occurs does not affect the probability of ***B*** occurring.

- Multiplication Rules
  - The **<u>multiplication rules</u>** can be used to find the probability of two or more events that occur in sequence.

  - When two events are independent, the probability of both occurring is:

$$\boldsymbol{P(A \text{ and } B) = P(A)P(B)}$$

5-7 Independent Events

- Example:
  - A box contains 3 red balls, 2 blue balls and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.
    - a. Selecting 2 blue balls

$$P(Blue\ and\ Blue) = P(Blue)P(Blue) = \left(\frac{2}{10}\right)\left(\frac{2}{10}\right) = \frac{4}{100} = \frac{1}{25}$$

    - b. Selecting 1 blue ball and then 1 white ball

$$P(Blue\ and\ White) = P(Blue)P(White) = \left(\frac{2}{10}\right)\left(\frac{5}{10}\right) = \frac{10}{100} = \frac{1}{10}$$

    - c. Selecting 1 red ball and then 1 blue ball

$$P(Red\ and\ Blue) = P(Red)P(Blue) = \left(\frac{3}{10}\right)\left(\frac{2}{10}\right) = \frac{6}{100} = \frac{3}{50}$$

5-7 Independent Events

- Example:
  - Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.

Let C denote red-green color blindness. Then

$$P(C \text{ and } C \text{ and } C) = P(C)P(C)P(C) = (0.09)(0.09)(0.09) = 0.000729$$

Hence, the rounded probability is 0.0007