

Hyperbolas with Center at the Origin

| | Horizontal Transverse Axis | Vertical Transverse Axis |
|------------------------|--|--|
| Graph | Figure 9.3.3 | Figure 9.3.4 |
| Equation | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a, b > 0$ | $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1, a, b > 0$ |
| Center | (0, 0) | (0, 0) |
| Foci | $(-c, 0), (c, 0), c = \sqrt{a^2 + b^2}$ | $(0, -c), (0, c), c = \sqrt{a^2 + b^2}$ |
| Vertices | $(-a, 0), (a, 0)$ | $(0, -a), (0, a)$ |
| Transverse axis | Segment of x -axis from $(-a, 0)$ to $(a, 0)$ | Segment of y -axis from $(0, -a)$ to $(0, a)$ |
| Asymptotes | The lines $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$ | The lines $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$ |

Hyperbolas with Center at (h, k)

| | Horizontal Transverse Axis | Vertical Transverse Axis |
|-----------------|---|---|
| Graph | Figure 9.3.9 | Figure 9.3.10 |
| Equation | $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1, a, b > 0$ | $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1, a, b > 0$ |
| Center | (h, k) | (h, k) |
| Foci | $(h - c, k), (h + c, k), c = \sqrt{a^2 + b^2}$ | $(h, k - c), (h, k + c), c = \sqrt{a^2 + b^2}$ |
| Vertices | $(h - a, k), (h + a, k)$ | $(h, k - a), (h, k + a)$ |
| Transverse axis | Segment of the line $y = k$ from $(h - a, k)$ to $(h + a, k)$ | Segment of the line $x = h$ from $(h, k - a)$ to $(h, k + a)$ |
| Asymptotes | The lines $y = \frac{b}{a}(x - h) + k$ and $y = -\frac{b}{a}(x - h) + k$ | The lines $y = \frac{a}{b}(x - h) + k$ and $y = -\frac{a}{b}(x - h) + k$ |

► Hyperbola with a horizontal transverse axis:

| | |
|------------------------|---|
| Equation | $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1, a, b > 0$ |
| Center | (h, k) |
| Foci | $(h - c, k), (h + c, k), c = \sqrt{a^2 + b^2}$ |
| Vertices | $(h - a, k), (h + a, k)$ |
| Transverse axis | Parallel to the x -axis between $(h - a, k)$ and $(h + a, k)$ |
| Asymptotes | $y = \frac{b}{a}(x - h) + k$ and $y = -\frac{b}{a}(x - h) + k$ |

► Hyperbola with a vertical transverse axis:

| | |
|------------------------|---|
| Equation | $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1, a, b > 0$ |
| Center | (h, k) |
| Foci | $(h, k - c), (h, k + c), c = \sqrt{a^2 + b^2}$ |
| Vertices | $(h, k - a), (h, k + a)$ |
| Transverse axis | Parallel to the y -axis between $(h, k - a)$ and $(h, k + a)$ |
| Asymptotes | $y = \frac{a}{b}(x - h) + k$ and $y = -\frac{a}{b}(x - h) + k$ |