Math 204

Homework 1.1

1) For each of the following ordinary differential equations state the order and determine whether the equation is linear or nonlinear, and indicate the independent and dependent variables.
   a) $3y'' + 2y' + 9y = 2\tan x$
   b) $\frac{dx}{dt} = \frac{x(2-3x)}{t(1-3x)}$
   c) $x^2(y''')^4 - 6xy' + 15y = \cos(x+y)$
   d) $\sqrt{1-\left(\frac{d^2y}{dx^2}\right)} - y = 0$

2) Determine whether the indicated function is an explicit solution to the given differential equation.
   a) $x = \cos 2t, \qquad x' + tx = \sin 2t$
   b) $y = e^{2x} - 3e^{-x}, \qquad y'' - y' - 2y = 0$

3) Determine whether the given relation is an implicit solution to the given differential equation.
   a) $x^2 + y^2 = 4 \qquad \frac{dy}{dx} = \frac{x}{y}$
   b) $e^{xy} + y = x - 1 \qquad \frac{dy}{dx} = \frac{e^{-xy}-y}{e^{-xy}+x}$

4) Show that $\varphi(x) = c_1 \sin x + c_2 \cos x$ is a solution to $y'' + y = 0$ for any choice of the constants c_1 and c_2. Thus $c_1 \sin x + c_2 \cos x$ is a two-parameter family of solutions to the differential equation.

5) Show that $\varphi(x) = c_1x^{-1} + c_2x + c_3x\ ln\ x + 4x^2$ is a solution to $x^3y''' + 2x^2y'' - xy' + y = 12x^2$ for any choice of the constants c_1 and c_2. Thus $c_1 \sin x + c_2 \cos x$ is a three-parameter family of solutions to the differential equation.

6) Find values of m so that the function $y = e^{mx}$ is a solution to $y'' + 6y' + 5y = 0$.

7) Verify that the pair of functions

$$x = \cos 2t + \sin 2t + \tfrac{1}{5}e^t,$$
$$y = -\cos 2t - \sin 2t - \tfrac{1}{5}e^t$$

   is a solution of the system of differential equations:

$$x'' = 4y + e^t$$
$$y'' = 4x - e^t$$