Math 311 all sections Fall 2013

You must solve question number one and any other one.

1. For the sequence $\{\frac{2n^2+3}{3n^2-n}\}_{n=1}^{\infty}$ find the limit and prove your answer using the ***definition***.

2. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $|f(x)-f(y)|\le C|x-y|$, for all $x,y\in\mathbb{R}$ and for some $C>0$. Let $\{a_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = a\in\mathbb{R}$.

(a) Prove that $\lim_{n\to\infty} f(a_n) = f(a)$.

(b) Is the sequence $\{\tan^{-1}(1+\frac{1}{n})\}$ convergent? If it is convergent find the limit.

3. Let $\{x_n\}$ be a convergent sequence and $\{y_n\}$ is a divergent sequence.

(a) Prove that $\{x_n+y_n\}$ is divergent.

(b) Is the sequence $\{(-1)^n+\frac{1}{n}\}$ convergent? If it is convergent find the limit.

4. Let $\{x_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} x_n = x\in\mathbb{R}^+$. Let $m\in\mathbb{Z}$.

(a) Prove that $\lim_{n\to\infty} x_n^m = x^m$.

(b) Prove that there exist $N\in\mathbb{N}$ such that if $n>N \Rightarrow \frac{x}{3}<x_n<3x$.

5. Let $\{x_n\}$ be a sequence of real numbers.

(a) Prove that there exist a sequence $\{m_n\}\subset\mathbb{Z}$ such that $\lim_{n\to\infty}[x_n-\frac{m_n}{n}]=0$.

(b) If $\lim_{n\to\infty} x_n = x$, prove that $\lim_{n\to\infty}\frac{m_n}{n}=x$.

6. Let $\{y_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} y_n = 0$.

(a) Let $\{x_n\}$ be a sequence of real numbers and $x\in\mathbb{R}$. Suppose that there is $N_1\in\mathbb{N}$ such that if $n>N_1\Rightarrow |x_n-x|\le y_n$. Prove that $\lim_{n\to\infty} x_n = x$.

(b) Suppose that $\{x_n\}$ is bounded sequence. Prove that $\lim_{n\to\infty} x_n y_n = 0$.

7. Let $x_1 = 1$ and $x_{n+1}=\sqrt{2x_n+3}$, for $n\ge 2$

(a) Prove that $1\le x_n\le 3$.

(b) Prove that $x_n\le x_{n+1}$.

(c) Prove that $\{x_n\}$ is convergent and find its limit.

8. Let $x_1=\frac{3}{2}$ and $x_{n+1}=2-\frac{1}{x_n}$, for $n\ge 2$

(a) Prove that $1<x_n<2$.

(b) Prove that $x_n>x_{n+1}$.

(c) Prove that $\{x_n\}$ is convergent and find its limit.

