Math 311 all sections Winter 2013

1. Let A be nonempty subset of $\mathbb{R}$. Let $\beta \in \mathbb{R}$. Then $\beta = \inf A$ if and only if β is a lower bound of A and for each $\epsilon > 0$ there exist $a_\epsilon \in A$ such that $a_\epsilon < \beta + \epsilon$.

2. Let A, B be two nonempty subsets of $\mathbb{R}$. Prove that $\inf(A + B) = \inf A + \inf B$

3. Let A be nonempty subset of $\mathbb{R}$, and $c \in \mathbb{R}$. If $c > 0$, prove that $\sup(cA) = c \sup A$.

4. Let A be nonempty subset of $\mathbb{R}$, and $c \in \mathbb{R}$. If $c < 0$, prove that $\sup(cA) = c \inf A$.

5. Let $0 < a < b$ and $b - a > 1$. Prove that there exist $n \in \mathbb{N}$ such that $a < n < b$.

6. Let $0 < a < b$. Prove that there exist $n \in \mathbb{N}$ such that $na \leq b < b(n + 1)$.

7. Let $A = \{x \in \mathbb{Q} | 3 \leq x^2 \leq 7\}$. Prove that $\sup A = \sqrt{7}$.

8. Let $A = \{x \in \mathbb{Q} | 3 \leq x^2 \leq 7\}$. Prove that $\inf A = -\sqrt{3}$.

9. Let $a \in \mathbb{R}$. Prove that for each $n \in \mathbb{N}$ there exist $r_n \in \mathbb{Q}$ such that $a - \frac{1}{n} < r_n < a + \frac{1}{n}$.

10. Let A, B be bounded two nonempty subsets of $\mathbb{R}$. If $A \subseteq B$, prove that $\sup A \leq \sup B$.

11. Let A, B be bounded two nonempty subsets of $\mathbb{R}$. If $A \subseteq B$, prove that $\inf B \leq \inf A$.

