



Homework One

Due date:

Department of Mathematics

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Math 311 all sections Winter 2013

1. Let $a, b \in \mathbb{R}$. If $0 < a < b$, prove that $a^2 < \sqrt{ab} < b^2$.
2. Let $a, b \in \mathbb{R}$. Prove that $\frac{(a+b)^2}{4} \leq \frac{a^2+b^2}{2}$.
3. Let $a, b \in \mathbb{R}$. If $0 < r < 1$ and $a < b$, prove that $a < ra + (1-r)b < b$.
4. Let $a, b \in \mathbb{R}$. Prove that $ab \leq \frac{(a+b)^2}{4}$.
5. Let $a \in \mathbb{R}$. Prove that $|a| = \sqrt{a^2}$.
6. Let $a, b \in \mathbb{R}$. Prove that $|a| \leq |b| \Leftrightarrow a^2 \leq b^2$.
7. Let $a, b \in \mathbb{R}$ and $b \neq 0$. Prove that $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$.
8. Let $a, b \in \mathbb{R}$. Prove that $|a+b| = |a| + |b|$ if and only if $ab > 0$.
9. Let $a \in \mathbb{R}$ and $a \neq 0$. Prove that $\frac{1}{\frac{1}{a}} = a$.
10. Let $a \in \mathbb{R}$. Prove that $-(-a) = a$.
11. Let $a \in \mathbb{R}$. Prove that $-1.a = -a$.
12. Prove that $(-1)(-1) = 1$.