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EMPIRICAL DETERMINATION OF THE APPARENT SHAPES AND ORIENTATIONS OF GALAXY CLUSTERS

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ABSTRACT

In this paper, an empirical iterative method is developed to specify the basic parameters that describe a galaxy cluster, such as the center of the cluster, the position angle of its projected major axis on the sky, and the length of its principal axes, as well as its apparent ellipticity. The procedure was applied to the Coma Cluster of galaxies, and it produced good values for these parameters.

Subject headings: galaxies: clustering

I. INTRODUCTION

The determination of the basic parameters that describe a galaxy cluster, such as the center of the cluster, the alignment of its projected major plane, the length of its prinicipal axes, and its apparent ellipticity, plays an essential role in extragalactic studies, particularly in those studies concerned with the clustering and orientation of galaxies (see, e.g., Gregory, Thompson, and Tifft 1981; MacGillivray *et al.* 1982). In the past, these parameters were obtained either by the visual inspection of photographic plates or by applying some analytic distributions such as the bivariate normal distribution of vector components or position coordinates (Carter and Metcalfe 1980; Trumpler and Weaver 1953). The latter method is an analytic iterative method which, when applied to the Coma Cluster, renders remarkably well the diminishing lengths of the cluster's principal axes. On the other hand, no observational techniques have been developed to specify such parameters with high accuracy.

In the present paper, an empirical iterative procedure will be introduced to evaluate the cluster's basic parameters accurately and to show as well whether galaxies are inclined to exist along a vivid and distinct line or whether they are distributed uniformly in all directions. In other words, the method depends on calculating the galaxy condensation relative to many position angles measured from north through east. The lengths of semimajor and semiminor axes are computed along the vivid and distinct line mentioned above and the direction perpendicular to it, respectively. Eventually, the apparent ellipticity is deduced from the principal axes.

As an example, the approach will be applied to the Coma Cluster of galaxies whose initial center is taken to be the point midway between the two giant ellipticals NGC 4874 and NGC 4889.

II. DATA

A region of 9.19 $\deg^2$ of sky centered on the Coma Cluster was examined using IIIaj plates taken with the UK 1.2 m Schmidt telescope. The exposure time was 100 minutes, and the plate grade is good, but the plate is quite dense. Although Schmidt plates can be used to detect objects down to a limiting magnitude $m_b = 21.5$, galaxies fainter than 19.0 mag were excluded from this paper to minimize the contribution of background objects. The filter used in the observation was GG 395.

Scans were made with the COSMOS measuring machine at the Royal Observatory of Edinburgh with a pixel size of 16 μm (equivalent $\approx 1''$ on the plate), and the COSMOS software was used to detect images above 7% of the night sky intensity level corresponding to the $\mu_j = 25$ mag arcsec^{-2} isophote. The software links pixels which belong to the same image and computes several parameters such as image centroid and isophotal magnitude.

The star/galaxy discrimination was done following techniques presented in detail by MacGillivray and Stobie (1985).

Since the main body of the Coma Cluster is located within a radius of 4 h_{50}^{-1} Mpc (Bahcall 1977), galaxies investigated in the present study are those included within a circle of this radius ($\sim 2.67\ h_{75}^{-1}$ Mpc) and with $m_b \leq 19.0$ mag. The number of such galaxies is 900.

III. DESCRIPTION OF THE METHOD

The method depends primarily on the trend of the cluster galaxies to be distributed along a main direction. Otherwise, it would be most difficult to determine the basic parameters. The procedure can be summarized in the following steps:

1. Estimate the initial values for the center of the cluster and its principal axes from the visual inspection of the photographic plate and obtain the corresponding data.

2. Calculate the actual center from the formulae

$$\bar{x} = \frac{1}{N} \sum_i x_i\ , \quad \bar{y} = \frac{1}{N} \sum_i y_i\ ,$$

where N is the number of galaxies and x_i and y_i are their coordinates.

3. Plot the relation $N(\theta)$ versus θ, where $N(\theta)$ is the number of galaxies within a strip of specific width (say 10% of the cluster's major axis) which passes through the cluster center. The position angle θ ranges from 0° to 180°. The cluster's projected plane is defined to be at an angle $\theta = \theta_{\max}$, where a single outstanding peak appears in the position angle distribution.

4. Once the direction of the projected major axis is obtained, the lengths of the semimajor and the semiminor axes are computed by projecting the coordinates of all galaxies in this direction and in the direction normal to it, respectively. The average of say $2n$ largest projections (with n on each side) represents the length of the semiaxis. It should be noted that if

TABLE 1

Initial and Final Values for Each Iteration of the Empirical Method

Iteration Number	Number of Galaxies	Initial Values XC (mm)	YC (mm)	a (h_{75}^{-1} Mpc)	b (h_{75}^{-1} Mpc)	Final Values θ_{max}	$\Delta\theta_{max}$	a	Δa	b	Δb	E	ΔE
1	900	171.65	187.90	2.68	2.68	78°36′	0°3′	2.25	0.174	2.15	0.138	0.0439	0.0126
2	868	174.35	185.70	2.25	2.15	79 23	0 18	2.09	0.097	1.99	0.108	0.0486	0.0076
3	834	175.18	184.71	2.09	1.99	81 17	0 24	...	...	...	...	...	...

n is large the lengths of the principal axes may be underestimated. On the other hand, if n is small the relative error in the projected ellipticity gets higher. Consequently, an optimum value of n is obtained by trial. In the case of the Coma Cluster, n was found to be 15.

5. The apparent ellipticity is then deduced from the formula

$$E = 1 - \frac{b}{a},$$

where a and b are the semimajor and semiminor axes, respectively.

6. The new data is computed using the latter values of

$$\bar{x},\ \bar{y},\ \theta_{max},\ a,\ \text{and}\ b\ .$$

7. Steps (2)–(6) are repeated until the values for each of the quantities $\bar{x}$, $\bar{y}$, θ_{max}, a, b, and E converge. The relative errors in these parameters should reduce with iterations.

IV. RESULTS AND ACCURACY

Table 1 demonstrates the results of applying the iterative method to the Coma Cluster. The table indicates both the initial and final values for each iteration. One can easily see that, by the third iteration, the values of the basic parameters converge. It is noteworthy that selection of good starting values of the cluster's center and principal axes accelerates the convergence appreciably. Figure 1, constructed using the latest

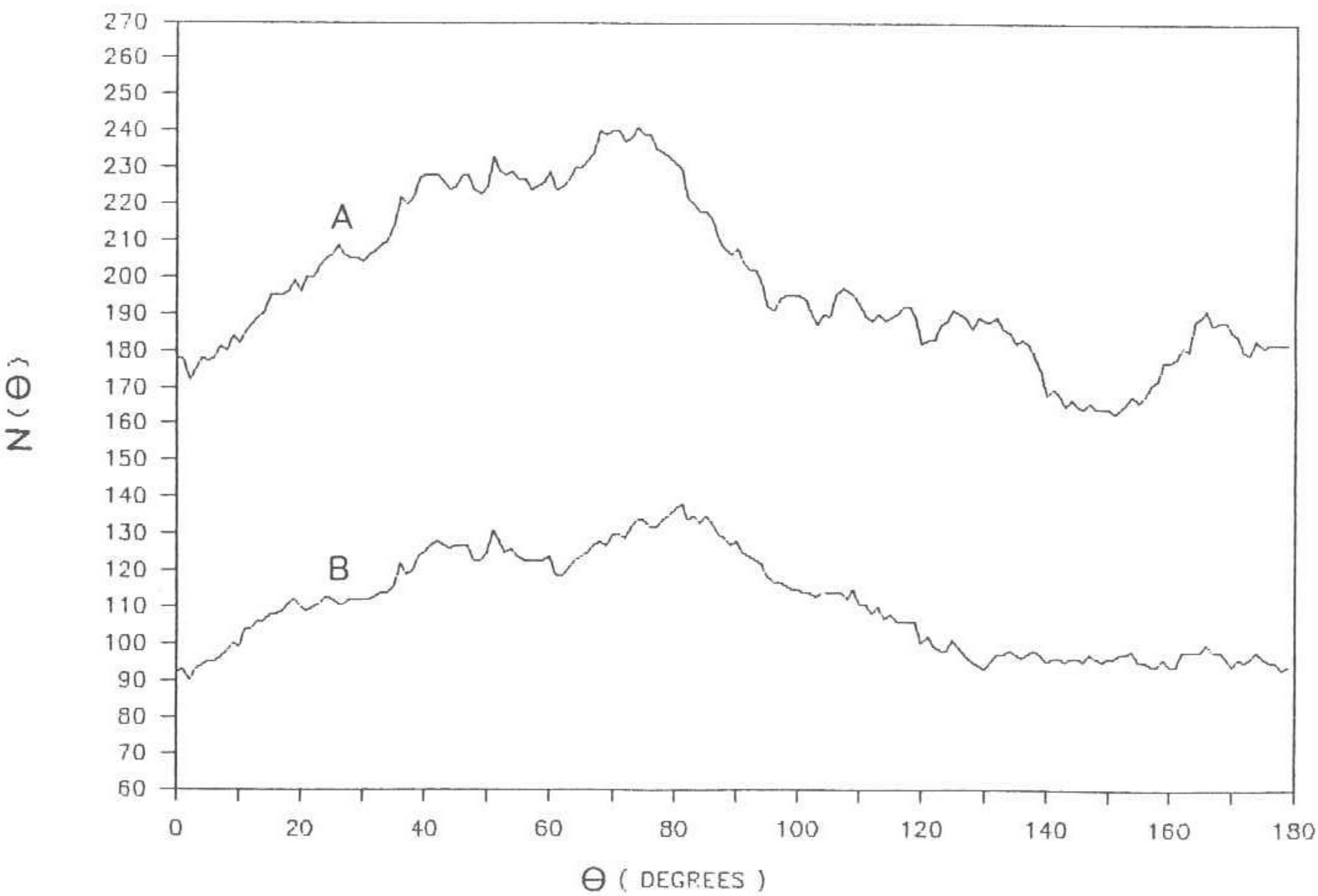

Fig. 1.—The frequency distribution of Coma galaxies as a function of position angle θ measured from north through east. The curve A represents all galaxies while the curve B refers to the bright ones.

values of the basic parameters, shows the frequency distribution of Coma galaxies as a function of position angle θ measured from north through east. The curve labeled A represents all galaxies, whereas the curve labeled B refers to the bright ones. The former curve exhibits a top which stretches out to cover a wide range. However, the latter one has a single dominant and distinct apex at position angle $81^\circ 17' \pm 24'$, since the contribution of background galaxies is significantly diminished by excluding the fainter half of the sample. The number of bright galaxies distributed within a strip of 0.45 h_{75}^{-1} Mpc width along the direction of position angle $81^\circ 17'$ is 138, which stands for $\sim 33\%$ of the total luminous galaxies.

Through applying the statistical tests presented by Hawley and Peebles (1975) to the bright galaxy distribution, a χ^2 test disclosed that the distribution is very highly nonrandom with confidence level greater than 99.5%. A Fourier transform also confirmed the same conclusion.

The errors in the values of basic parameters were illustrated in Table 1 and were estimated as follows:

1. As for the semimajor and semiminor axes, the errors Δa and Δb were expressed by the root mean square of the values of 30 projections which were referred to in the previous section.

2. The error in the apparent ellipticity ΔE was determined in terms of Δa and Δb according to the formula

$$\Delta E = \frac{\partial E}{\partial a} \Delta a + \frac{\partial E}{\partial b} \Delta b \, .$$

3. The error in the position angle of the projected major axis was obtained by investigating the distribution of bright galaxies in a range of 4° around the apex. The graduation unit in this distribution was taken to be 1′ instead of 1° as in the original distribution B. Thus, the angular interval wherein the maximum number of galaxies exists could easily be evaluated. θ_{max} is assumed to be the central value of this interval.

In conclusion, the empirical iterative method was developed to determine the basic parameters that describe a galaxy cluster, including the center of the cluster, the position angle of its projected major axis on the sky, and the length of its principal axes, as well as its apparent ellipticity. The procedure was applied to the Coma Cluster of galaxies and it produced good values for these parameters. It will be interesting to apply this method to other clusters of galaxies, particularly those of linear and flat (L, F) types in the Rood-Sastry (Rood and Sastry 1971) classification system.

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