Math 251

Assignment 3

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All exercises from 1- 10 can be found in the book of Kenneth Rosen, seventh edition.

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1. Determine the truth value of each of these statements if the domain for all variables consists of: (i) all integers.
(ii) all real numbers.

(a) $\forall n \exists m (n^2 < m)$.
(b) $\exists n \forall m (n < m^2)$.
(c) $\forall n \exists m (n + m = 0)$.
(d) $\exists n \forall m (nm = m)$.
(e) $\exists n \exists m (n^2 + m^2 = 5)$.
(f) $\exists n \exists m (n^2 + m^2 = 6)$.
(g) $\exists n \exists m (n + m = 4 \wedge n - m = 1)$
(h) $\exists n \exists m (n + m = 4 \wedge n - m = 2)$
(i) $\forall n \forall m \exists p (p = (m + n)/2)$
(j) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$
(k) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

2. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

(a) $\forall v \forall y P(x, y)$.
(b) $\exists x \exists y P(x, y)$.
(c) $\exists x \forall y P(x, y)$.
(d) $\forall y \exists x P(x, y)$.

3. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

(a) $\exists y \exists x P(x, y)$
(b) $\forall x \exists y P(x, y)$
(c) $\exists y (Q(y) \wedge \forall x \neg R(x, y))$
(d) $\exists y (\exists x R(x, y) \vee \forall x S(x, y))$
(e) $\exists y (\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$
(f) $\forall x \exists \forall z T(x, y, z)$
(g) $\forall x \exists y (P(x, y) \vee \forall \exists Q(x, y))$
(h) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$
(i) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

4. Find a common domain for the variables x, y, z, and w for which the statement $\forall x \forall y \forall z \exists w((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$ is true and another common domain for these variables for which it is false.

5. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a) $\forall x \forall y(x^2 = y^2 \rightarrow x = y)$.
(b) $\forall x \exists y(y^2 = x)$.
(c) $\forall x \forall y(xy \geq x)$.
(d) $\forall x \exists y(x = 1/y)$.
(e) $\forall x \exists y(y^2 - x < 100)$.
(f) $\forall x \forall y(x^2 \neq y^3)$.

6. Determine the truth value of the statement $\forall x \exists y(xy = 1)$ if the domain for the variables consists of.

a) the nonzero real numbers.
b) the nonzero integers.
c) the positive real numbers.

7. Determine the truth value of the statement $\exists x \forall y(xy^2)$ if the domain for the variables consists of

a) the positive real numbers.
b) the integers.
c) the nonzero real numbers.

8. Show that the two statements $\neg \exists x \forall y P(x, y)$ and $\forall x \exists y \neg P(x, y)$, where both quantifiers over the first variable in $P(x, y)$ have the same domain, and both quantifiers over the second variable in $P(x, y)$ have the same domain, are logically equivalent.

9. Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x \exists y(P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.

10. Show that $\forall x P(x) \vee \exists x Q(x)$ is equivalent to $\forall x \exists y(P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain.

11. Find the negation of the following statements:

(a). Any rectangular triangle has a right angle.

(b) For any integer $x \in \mathbb{Z}$, there exists an integer $y \in \mathbb{Z}$ such that, for any $z \in \mathbb{Z}$, the inequality $z < x$ implies $z < x + 1$.

(c) $\forall \epsilon > 0 \forall x \in \mathbb{R} \exists \delta > 0 \forall y \in \mathbb{R}(|y - x| < \delta \to |f(y) - f(x)| < \epsilon)$.

(d) $\forall \epsilon > 0 \exists N \in \mathbb{N}[(x \in \mathbb{Z} \wedge \forall n \geq N) \to |f_n(x) - f(x)| < \epsilon]$.

(e) $\forall x \in \mathbb{R} \forall y \in \mathbb{R}[x = y \leftrightarrow (\forall \epsilon > 0 \to |x - y| < \epsilon)]$.