Math 251

Assignment 3

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All exercises from 3 -19 can be found in the book of Kenneth Rosen, sixth edition.

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1. Simplify the following propositions:

(a) $\neg(p \vee \neg(p \wedge q))$.
(b) $[(p \to q) \wedge \neg q] \to \neg p$.

2. Which of the following are equivalent to $\neg(p \to r) \to \neg q$ (there may be more than one or none).

(a) $\neg(p \to r) \vee q$.
(b) $(p \wedge \neg r) \vee q$.
(c) $(\neg p \to \neg r) \vee q$.
(d) $q \to (p \to r)$.
(e) $\neg q \to (\neg p \to \neg r)$.
(f) $\neg q \to (\neg p \vee r)$.
(g) $\neg q \to \neg(p \to r)$.

3. Let $P(x)$ denote the statement "$x \leq 4$". What are the truth values of:

(a) $P(0)$?
(b) $P(4)$?
(c) $P(6)$?

4. Let $P(x)$ be the statement "the word x contains the letter a". What are the truth values of

(a) $P(\text{orange})$?
(b) $P(\text{lemon})$?
(c) $P(\text{true})$?
(d) $P(\text{false})$?

5. Let $Q(x, y)$ be the statement " x is the capital of y". What are the truth values of

(a) $Q(\text{Jeddah, Saudi Arabia})$.
(b) $Q(\text{Dubai, UAE})$.
(c) $Q(\text{Kuwait, Kuwait})$.

6. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

(a) $\exists x P(x)$.
(b) $\forall x P(x)$.
(c) $\exists x \neg P(x)$.
(d) $\forall x \neg P(x)$.

7. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language $C++$." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows $C++$.
b) There is a student at your school who can speak Russian but who doesn't know $C++$.
e) Every student at your school either can speak Russian or knows $C++$.
d) No student at your school can speak Russian or knows $C++$.

8. Let $P(x)$ be the statement "$x+1 > 2x$." If the domain consists of the integers, what are the truth values?

(a) $P(0)$.
(b) $P(1)$.
(c) $P(2)$.
(d) $P(-1)$.
(e) $\exists x P(x)$.
(f) $\forall x P(x)$.

9. Determine the truth value of each of these statements if the domain consists of all integers.

(a) $\forall n(n+1 > n)$.
(b) $\exists n(2n = 3n)$.
(c) $\exists n(n = -n)$.

(d) $\forall n(n^2 \geq n)$.

9. Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

(a) $\exists x P(x)$
(b) $\forall x P(x)$
(c) $\neg\exists x P(x)$
(d) $\neg\forall x P(x)$
(e) $\forall x((x \neq 3) \to P(x)) \vee \exists x \neg P(x)$

10. For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

(a) Everyone is studying discrete mathematics.
(b) Everyone is older than 21 years.
(c) Every two people have the same mother.
(d) No two different people have the same grandmother.

11. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.
b) Everyone in your class is friendly.
c) There is a person in your class who was not born in California.
d) A student in your class has been in a movie.
e) No student in your class can solve quadratic equations.

12. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) No one is perfect.
b) Not everyone is perfect.
c) All your friends are perfect.
d) At least one of your friends is perfect.
e) Everyone is your friend and is perfect.

f) Not everybody is your friend or someone is not perfect.

13. Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1$, or $2, y = 0$ or 1, and $z = 0$ or 1. Write out these propositions using disjunctions and conjunctions.

(a) $\forall y Q(0, y, 0)$
(b) $\exists x Q(x, 1, 1)$
(c) $\exists z \neg Q(0, 0, z)$
(d) $\exists x \neg Q(x, 0, 1)$.

14. Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the words "It is not the case that.")

a) Some old dogs can learn new tricks.
b) No rabbit knows calculus.
e) Every bird can fly.
d) There is no dog that can talk.
e) There is no one in this class who knows French and Russian.

15. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a) $\forall x(x^2 \geq x)$
(b) $\forall(x > 0 \vee x < 0)$
(c) $\forall x(x = 1)$.

16. Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

(a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$
(b) $\forall p B(p) \rightarrow \exists j Q(j)$
(c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$
(d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists L(j)$.

17. Determine whether $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are logically equivalent. Justify your answer.

18. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

(a) $(\forall x P(x)) \land A \equiv \forall x(P(x) \land A)$
(b) $(\exists x P(x)) \land A \equiv \exists x(P(x) \land A)$.

19. What are the truth values of these statements?

(a) $\exists! x P(x) \to \exists x P(x)$
(b) $\forall x P(x) \to \exists! x P(x)$
(c) $\exists! x \neg P(x) \to \neg \forall x P(x)$.