MATH 251

Assignment 1: Propositional Logic

The due date for this assignment is 10 September 2015.

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

   - Miami is the capital of Florida.
   - $2 + 3 = 5$.
   - $5 + 7 = 10$.
   - $x + 2 = 11$.
   - Answer this question.

2. What is the negation of each of these propositions?

   - Mei has an MP3 player.
   - There is no pollution in New Jersey.
   - $2 + 1 = 3$.
   - The summer in Maine is hot and sunny.

3. Let p be "It is cold" and q be "It is raining". Give a simple verbal sentence which describe each of the following:

   (a) $(p \wedge \sim q) \longrightarrow p$

   (b) $p \leftrightarrow \sim q$

4. Let p "He is tall" and q be "He is handsome". Write each of the following statement using p and q.

   - It is false that he is short or handsome
   - He is tall, or he is short and handsome
   - It is not true that he is short or not handsome

5. Determine the truth value of each of the following:

- It is not true that $2+2=5$ if and only if $4+4=10$
- It is not true that $1+1=3$ or $2+1=3$
- It is false that if Paris is in England then London is in France

6. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

   a) $\neg q$ b) $p \wedge q$ c) $\neg p \vee q$

   d) $p \to \neg q$ e) $\neg q \to p$ f) $\neg p \to \neg q$

   g) $p \leftrightarrow \neg q$ h)$\neg p \wedge (p \vee \neg q)$

7. Let p and q be the propositions

   p : It is below freezing.

   q : It is snowing.

   Write these propositions using p and q and logical connectives (including negations).

   - It is below freezing and snowing.
   - It is below freezing but not snowing.
   - It is not below freezing and it is not snowing.
   - If it is below freezing, it is also snowing.
   - Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
   - That it is below freezing is necessary and sufficient for it to be snowing.

8. Determine whether these biconditionals are true or false.

   - $2+2=4$ if and only if $1+1=2$.
   - $1+1=3$ if and only if monkeys can fly.
   - $0>1$ if and only if $2>1$.

9. Determine whether each of these conditional statements is true or false.

   - If $1+1=2$, then $2+2=5$.
   - If $1+1=3$, then $2+2=5$.
   - If monkeys can fly, then $1+1=3$.

10. State the converse, contrapositive, and inverse of each of these conditional statements.

    - If it snows today, I will ski tomorrow.
    - I come to class whenever there is going to be a quiz.
    - A positive integer is a prime only if it has no divisors other than 1 and itself.

11. How many rows appear in a truth table for each of these compound propositions?

- $p \to \neg p$
- $(p \vee \neg r) \wedge (q \vee \neg s)$
- $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
- $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

12. Construct a truth table for each of these compound propositions.

- $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
- $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
- $(p \to q) \vee (\neg p \to q)$
- $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

13. Show that each of these conditional statements is a tautology or not a tautology by using truth tables.

- $[\neg p \wedge (p \vee q)] \to q$
- $(\neg q \wedge (p \to q)) \to \neg p$
- $[p \wedge (p \to q)] \to q$
- $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

14. Simplify each of the following propositions:

- $\sim (p \vee \sim q)$.
- $\sim (\sim p \wedge \sim q)$.
- $\sim (p \vee q) \vee (\sim p \wedge q)$.

15. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

16. Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

17. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

18. Find the dual of each of these compound propositions.

- $p \wedge \neg q \wedge \neg r$
- $(p \wedge q \wedge r) \vee s$
- $(p \vee F) \wedge (q \vee T)$