Chapter 6: Short Circuit Studies - Symmetrical Faults

Introduction

Short circuits occur in power system due to various reasons like, equipment failure, lightning strikes, falling of branches or trees on the transmission lines, switching surges, insulation failures and other electrical or mechanical causes. All these are collectively called **faults** in power systems.

A fault usually results in high current flowing through the lines and if adequate protection is not taken, may result in damages in the power apparatus.

In this chapter we shall discuss the effects of **symmetrical faults** on the system. Here the term symmetrical fault refers to those conditions in which all three phases of a power system are grounded at the same point. For this reason the symmetrical faults sometimes are also called three-line-to-ground (3LG) faults.

Section I: Transients in R-L Circuits

- <u>**DC Sourse**</u>
- <u>**AC Sourse**</u>
- <u>**Fault in an AC Circuit**</u>

Transients in R-L Circuits

In this section we shall consider transients in a circuit that contains a resistor and inductor (R - L circuit). Consider the circuit shown in Fig. 6.1 that contains an ideal source (v_s), a resistor (***R***), an inductor (***L***) and a switch (***S***). It is assumed that the switch is open and is closed at an instant of time t = 0. This implies that the current i is zero before the closing of the switch. We shall first discuss the effect of closing the switch on the line current *(i*) when the source is dc. Following this we shall study the effect when the source is ac and will show that the shape of the transient current changes with the changes in the phase of the source voltage waveform at the instant of closing the switch.

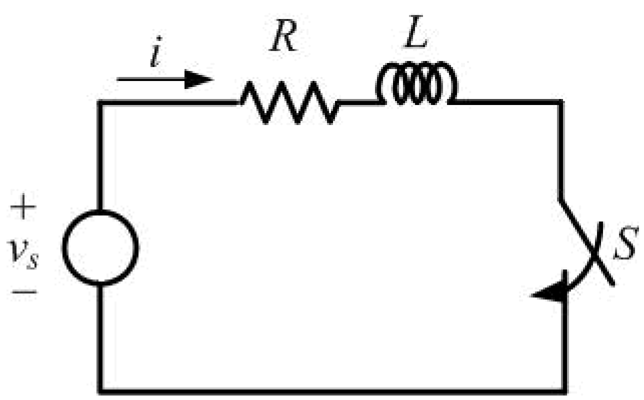

Fig 6.1 A Simple R - L Circuit

DC Source

Let us assume that the source voltage is dc and is given by $v_s = V_{dc}$. Then the line current is given by the differential equation

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v_s \tag{6.1}$$

$$i(t) = e^{-Rt/L}i(0) + \frac{1}{L}\int_0^t e^{-(R/L)(t-\tau)}v_s(\tau)d\tau \tag{6.2}$$

The solution of the above equation is written in the form

Since the initial current i (0) = 0 and since v_s (τ) = V_{dc} for 0 £ t < ¥ , we can rewrite the above equation

$$i(t) = \frac{V_{dc}}{R}\left(1 - e^{-Rt/L}\right) = \frac{V_{dc}}{R}\left(1 - e^{-t/T}\right) \tag{6.3}$$

as

where $T = L / R$ is the time constant of the circuit.

Let us assume R = 1Ω , L = 10 mH and V_{dc}= 100 V. Then the time response of the current is as shown in Fig. 6.2. It can be seen that the current reaches at steady state value of 100 A. The time constant of the circuit is 0.01 s. This is defined by the time in which the current i (t) reaches 63.2% of its final value and

$$\frac{di}{dt} = \frac{V_{dc}}{RT}e^{-t/T}\bigg|_{t=0} = \frac{V_{dc}}{RT} = 10^4 \tag{6.4}$$

is obtained by substituting t = T. Note that the slope of the curve is given by

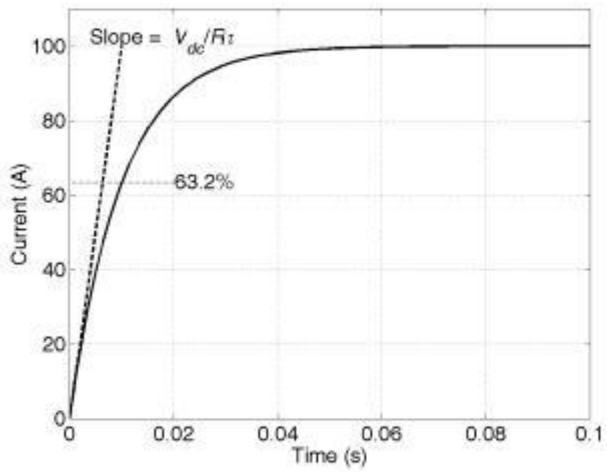

Fig 6.2 Current in the R-L circuit when the source is dc

AC Source

The current response remains unchanged when the voltage source is dc. This however is not the case

$$v_s = \sqrt{2}V_m \sin(\omega t + \alpha) \tag{6.5}$$

when the circuit is excited by an ac source. Let us assume that the source voltage is now given by where α is the phase angle of the applied voltage. We shall show that the system response changes with a change in α .

$$i(t) = i_{ac}(t) + i_{dc}(t) \tag{6.6}$$

$$i_{ac} = \frac{\sqrt{2}V_m}{Z}\sin(\omega t + \alpha - \theta) \text{ A} \tag{6.7}$$

$$i_{dc} = \frac{\sqrt{2}V_m}{Z}\sin(\alpha - \theta)e^{-t/T} \text{ A} \tag{6.8}$$

The solution of (6.2) for the source voltage given in (6.5) is

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The system response for V_m = 100 V and α = 45° is shown in Fig. 6.3. In this figure both i_{ac} and i_{dc} are also shown. It can be seen that i_{ac} is the steady state waveform of the circuit, while i_{dc} dies out once the initial transient phase is over. Fig. 6.4 shows the response of the current for different values of a . Since the current is almost inductive, it can be seen that the transient is minimum when α = 90° , i.e., the circuit is switched on almost at the zero-crossing of the current. On the other hand, the transient is maximum when α = 0° , i.e., almost at the peak of the current.

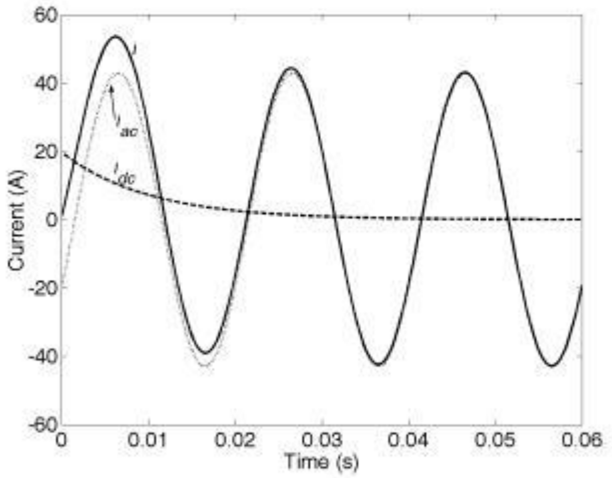

Fig 6.3 Transient in current and its ac and dc components at the instant of switch closing

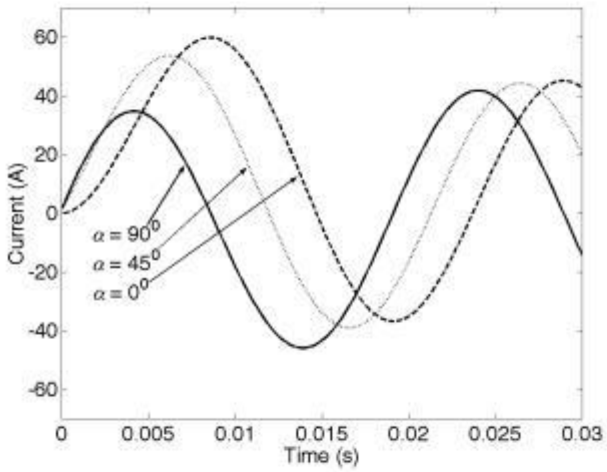

Fig 6.4 Transient in current for different values of α

Fault in an AC Circuit

Now consider the single-phase circuit of Fig. 6.5 where V_s = 240 V (rms), the system frequency is 50 Hz, R = 0.864 Ω, L = 11 mH (ωL = 3.46 Ω) and the load is R-L comprising of an 8.64 W resistor and a 49.5 mH inductor (ωL = 15.55Ω). With the system operating in the steady state, the switch S is suddenly closed creating a short circuit. The current (i) waveform is shown in Fig. 6.6. The current phasor before the short circuit occurs is

$$I = \frac{240}{9.504 + j19.01} = 5.05 - j10.10 = 11.29\angle -63.43^\circ \text{ A}$$

This means that the pre-fault current has a peak value of 15.97 A.

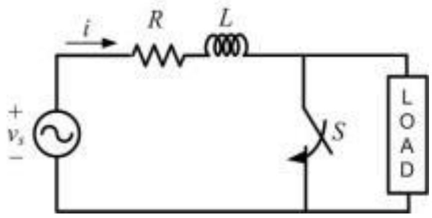

Fig. 6.5 A single-phase circuit in which a source supplies a load through a source impedance.

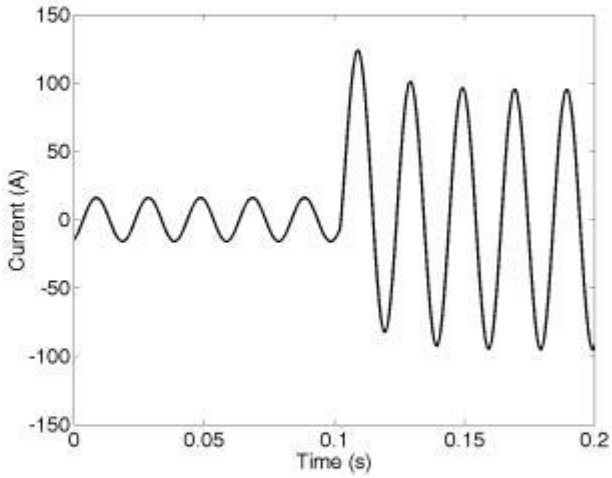

Fig 6.6 The current waveform of the circuit of Fig 6.5 before and after the closing of the switch S

Once the fault occurs and the system is allowed to reach the steady state, the current phasor is given by

$$I = \frac{240}{0.864 + j3.46} = 16.34 - j65.36 = 67.37\angle -75.96°$$

This current has a peak value of 95.28 A. However it can be seen that the current rises suddenly and the first peak following the fault is 124 A which is about 30% higher than the post-fault steady-state value. Also note that the peak value of the current will vary with the instant of the occurrence of the fault. However the peak value of the current is nearly 8 times the pre-fault current value in this case. In general, depending on the ratio of source and load impedances, the faulted current may shoot up anywhere between 10 and 20 times the pre-fault current.

Short Circuit in an Unloaded Synchronous Generator

Fig. 6.7 shows a typical response of the armature current when a three-phase symmetrical short circuit occurs at the terminals of an unloaded synchronous generator.

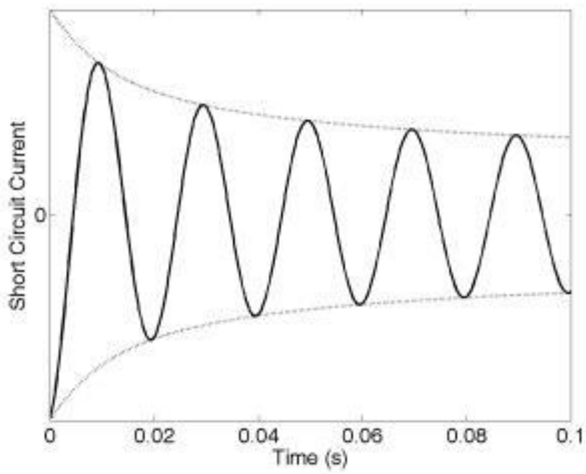

Fig. 6.7 Armature current of a synchronous generator as a short circuit occurs at its terminals.

It is assumed that there is no dc offset in the armature current. The magnitude of the current decreases

$$i_f(t) = \sqrt{2}V_t\left[\left(\frac{1}{X_d''}-\frac{1}{X_d'}\right)e^{-t/T_d''} + \left(\frac{1}{X_d'}-\frac{1}{X_d}\right)e^{-t/T_d'} + \frac{1}{X_d}\right]\sin\left(\omega t+\alpha-\pi/2\right) \tag{6.9}$$

exponentially from a high initial value. The instantaneous expression for the fault current is given by

where V_t is the magnitude of the terminal voltage, α is its phase angle and

X_d'' is the direct axis subtransient reactance
X_d' is the direct axis transient reactance
X_d is the direct axis synchronous reactance

with $X_d'' < X_d' < X_d$. The time constants are

T_d'' is the direct axis subtransient time constant
T_d' is the direct axis transient time constant

In the expression of (6.9) we have neglected the effect of the armature resistance hence α = π/2. Let us

$$I_f(0) = I_f'' = \frac{V_t}{X_d''} \tag{6.10}$$

assume that the fault occurs at time t = 0. From (6.9) we get the rms value of the current as

which is called the **subtransient fault current**. The duration of the subtransient current is dictated by the time constant T_d'' . As the time progresses and $T_d'' < t < T_d'$, the first exponential term of (6.9) will start decaying and will eventually vanish. However since t is still nearly equal to zero, we have the following

$$I_f' = \frac{V_t}{X_d'} \tag{6.11}$$

rms value of the current

This is called the **transient fault current**. Now as the time progress further and the second exponential

$$I_f = \frac{V_t}{X_d} \tag{6.12}$$

term also decays, we get the following rms value of the current for the sinusoidal steady state

In addition to the ac, the fault currents will also contain the dc offset. Note that a symmetrical fault occurs when three different phases are in three different locations in the ac cycle. Therefore the dc offsets in the three phases are different. The maximum value of the dc offset is given by

$$i_{dc}^{\max} = \sqrt{2} I_f'' e^{-t/T_A} \quad (6.13)$$

where T_A is the armature time constant.

Section III: Symmetrical Fault in a Power System

- **Calculation of Fault Current Using Impedance Diagram**
- **Calculation of Fault Current Using Z_{bus} Matrix**

Calculation of Fault Current Using Impedance Diagram

Let us first illustrate the calculation of the fault current using the impedance diagram with the help of the following examples.

Example 6.1

Consider the power system of Fig. 6.8 in which a synchronous generator supplies a synchronous motor. The motor is operating at rated voltage and rated MVA while drawing a load current at a power factor of 0.9 (lagging) when a three phase symmetrical short circuit occurs at its terminals. We shall calculate the fault current that flow from both the generator and the motor.

We shall choose a base of 50 MVA, 20 kV in the circuit of the generator. Then the motor synchronous reactance is given by

$$X_m'' = 0.2 \times \frac{50}{25} = 0.4 \text{ per unit}$$

Also the base impedance in the circuit of the transmission line is

$$Z_{base} = \frac{66^2}{50} = 87.12 \; \Omega$$

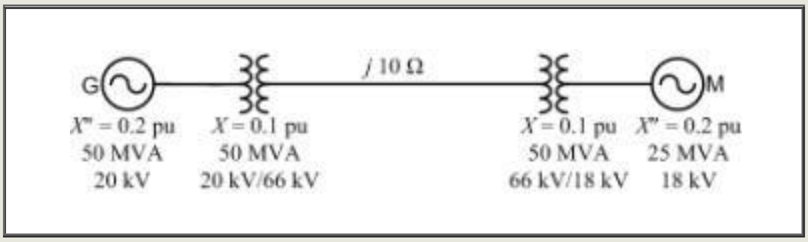

Fig. 6.8 A generator supplying a motor load though a transmission line.

Therefore the impedance of the transmission line is

$$X_{line} = j\frac{10}{87.12} = j0.1148 \text{ per unit}$$

The impedance diagram for the circuit is shown in Fig. 6.9 in which the switch *S* indicates the fault.

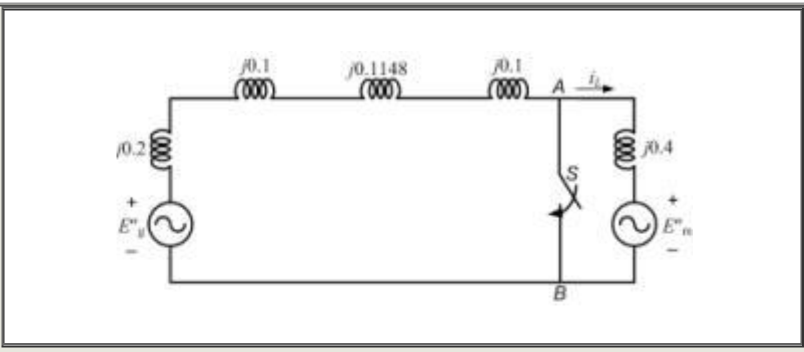

Fig. 6.9 Impedance diagram of the circuit of Fig. 6.8.

The motor draws a load current at rated voltage and rated MVA with 0.9 lagging power factor. Therefore

$$i_L = 1\angle -\cos^{-1}(0.9) = 0.9 - j0.4359 \text{ per unit}$$

Then the subtransient voltages of the motor and the generator are

$$E''_m = 1.0 - j0.4 \times i_L = 0.8256 - j0.36 \text{ per unit}$$

$$E''_g = 1.0 + j0.5148 \times i_L = 1.2244 + j0.4633 \text{ per unit}$$

Hence the subtransient fault currents fed by the motor and the generator are

$$I''_m = \frac{E''_m}{j0.4} = -0.9 - j2.0641 \text{ per unit}$$

$$I''_g = \frac{E''_g}{j0.5148} = 0.9 - j2.3784 \text{ per unit}$$

and the total current flowing to the fault is

$$I''_f = I''_g + I''_m = -j4.4425 \text{ per unit}$$

Note that the base current in the circuit of the motor is

$$I_{base} = \frac{50 \times 10^3}{\sqrt{3} \times 18} = 1603.8 \text{ A}$$

Therefore while the load current was 1603.8 A, the fault current is 7124.7 A.

Example 6.2

We shall now solve the above problem differently. The Thevenin impedance at the circuit between the terminals *A* and *B* of the circuit of Fig. 6.9 is the parallel combination of the impedances *j* 0.4 and *j* 0.5148. This is then given as

$$Z_{th} = j\frac{0.4 \times 0.5148}{0.4 + 0.5148} = j0.2251 \text{ per unit}$$

Since voltage at the motor terminals before the fault is 1.0 per unit, the fault current is

$$I''_f = \frac{1.0}{Z_{th}} = -j4.4425 \text{ per unit}$$

If we neglect the pre-fault current flowing through the circuit, then fault current fed by the motor and the generator can be determined using the current divider principle, i.e.,

$$I''_{m0} = \frac{I''_f}{j0.9148} \times j0.5148 = -j2.5 \text{ per unit}$$

$$I''_{g0} = \frac{I''_f}{j0.9148} \times j0.4 = -j1.9425 \text{ per unit}$$

If, on the other hand, the pre-fault current is not neglected, then the fault current supplied by the motor and the generator are

$$I''_m = I''_{m0} - I_L = -0.9 - j2.0641 \text{ per unit}$$

$$I''_g = I''_{g0} + I_L = 0.9 - j2.3784 \text{ per unit}$$

Calculation of Fault Current Using Z_{bus} Matrix

Consider the circuit of Fig. 3.3 which is redrawn as shown in Fig. 6.10.

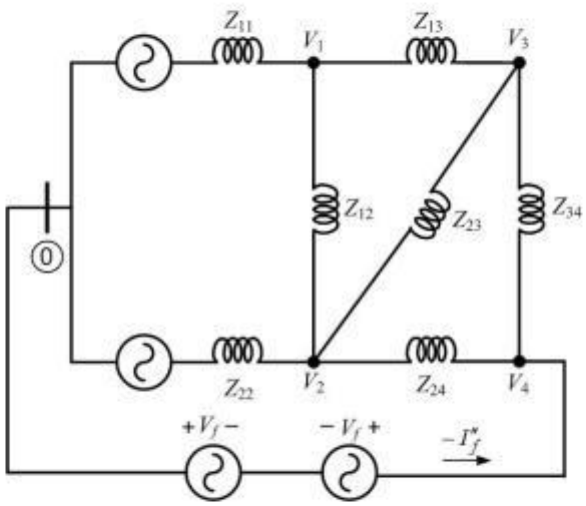

Fig. 6.10 Network depicting a symmetrical fault at bus-4.

We assume that a symmetrical fault has occurred in bus-4 such that it is now connected to the reference bus. Let us assume that the pre-fault voltage at this bus is V_f . To denote that bus-4 is short circuit, we add two voltage sources V_f and - V_f together in series between bus-4 and the reference bus. Also note that the subtransient fault current I''_f flows from bus-4 to the reference bus. This implies that a current that is equal to - I''_f is injected into bus-4. This current, which is due to the source - V_f will flow through the various branches of the network and will cause a change in the bus voltages. Assuming that the two sources and V_f are short circuited. Then - V_f is the only source left in the network that injects a current - I''_f into bus-4. The voltages of the different nodes that are caused by the voltage - V_f and the current - I''_f

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ -V_f \end{bmatrix} = Z_{bus} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I''_f \end{bmatrix} \tag{6.14}$$

are then given by

where the prefix Δ indicates the changes in the bus voltages due to the current - I''_f .

$$V_f = Z_{44} I''_f \tag{6.15}$$

From the fourth row of (6.14) we can write

$$\Delta V_i = -Z_{i4} I''_f = -\frac{Z_{i4}}{Z_{44}} V_f, \quad i = 1,2,3 \tag{6.16}$$

Combining (6.14) and (6.15) we get

We further assume that the system is unloaded before the fault occurs and that the magnitude and phase angles of all the generator internal emfs are the same. Then there will be no current circulating anywhere in the network and the bus voltages of all the nodes before the fault will be same and equal to V_f . Then

$$V_i = V_f + \Delta V_i = \left(1 - \frac{Z_{i4}}{Z_{44}}\right) V_f, \quad i = 1, \cdots, 4 \tag{6.17}$$

the new altered bus voltages due to the fault will be given from (6.16) by

Example 6.3

Let us consider the same system as discussed in Example 3.1 except that we assume that the internal voltages of both the generators are equal to 1.0 < 0° . Then the current injected in both bus-1 and 2 will be given by 1.0/ j 0.25 = - j 4.0 per unit. We therefore get the pre-fault bus voltages using the Z_{bus} matrix given in Example 3.1 as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = j \begin{bmatrix} 0.1531 & 0.0969 & 0.1264 & 0.1133 \\ 0.0969 & 0.1531 & 0.1236 & 0.1367 \\ 0.1264 & 0.1236 & 0.2565 & 0.1974 \\ 0.1133 & 0.1367 & 0.1974 & 0.3926 \end{bmatrix} \begin{bmatrix} 4\angle -90^\circ \\ 4\angle -90^\circ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

Now the altered bus voltages for a symmetrical fault in bus-4 are given from (6.17) as

$$V_1 = 1 - \frac{0.1133}{0.3926} = 0.7114$$ per unit

$$V_2 = 1 - \frac{0.1367}{0.3926} = 0.6518$$ per unit

$$V_3 = 1 - \frac{0.1974}{0.3926} = 0.4972$$ per unit

$$V_4 = 1 - \frac{0.3926}{0.3926} = 0$$ per unit

Also since the Thevenin impedance looking into the network at bus-4 is Z_{44} (see Section 3.4), the subtransient fault current flowing from bus-4 is

$$I_f'' = \frac{1}{j0.3926} = -j2.5471$$ per unit

Section IV: Circuit Breaker Selection

A typical circuit breaker operating time is given in Fig. 6.11. Once the fault occurs, the protective devices get activated. A certain amount of time elapses before the protective relays determine that there is overcurrent in the circuit and initiate trip command. This time is called the **detection time**. The contacts of the circuit breakers are held together by spring mechanism and, with the trip command, the spring mechanism releases the contacts. When two current carrying contacts part, a voltage instantly appears at the contacts and a large voltage gradient appears in the medium between the two contacts. This voltage gradient ionizes the medium thereby maintaining the flow of current. This current generates extreme heat and light that is called electric arc. Different mechanisms are used for elongating the arc such that it can be cooled and extinguished. Therefore the circuit breaker has to withstand fault current from the instant of initiation of the fault to the time the arc is extinguished.

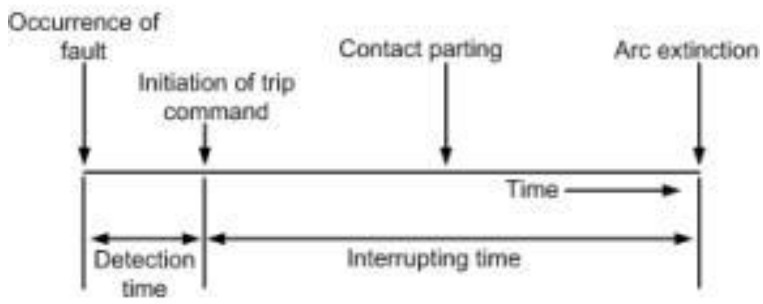

Fig. 6.11 Typical circuit breaker operating time.

Two factors are of utmost importance for the selection of circuit breakers. These are:

- The maximum instantaneous current that a breaker must withstand and
- The total current when the breaker contacts part.

In this chapter we have discussed the calculation of symmetrical subtransient fault current in a network. However the instantaneous current following a fault will also contain the dc component. In a high power circuit breaker selection, the subtransient current is multiplied by a factor of 1.6 to determine the rms value of the current the circuit breaker must withstand. This current is called the **momentary current** . The **interrupting current** of a circuit breaker is lower than the momentary current and will depend upon the speed of the circuit breaker. The interrupting current may be asymmetrical since some dc component may still continue to decay.

Breakers are usually classified by their nominal voltage, continuous current rating, rated maximum voltage, K -factor which is the voltage range factor, rated short circuit current at maximum voltage and operating time. The K -factor is the ratio of rated maximum voltage to the lower limit of the range of the operating voltage. The maximum symmetrical interrupting current of a circuit breaker is given by K times the rated short circuit current.