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Some Modifications of Adomian Decomposition Method Applied to Nonlinear System of Fredholm Integral Equations of the Second Kind

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Abstract

In this paper, some modifications of Adomian decomposition method apply for solving system of nonlinear Fredholm integral equations of the second kind. We present some numerical example to show the efficiency of each method.

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Keywords: Adomian decomposition method, nonlinear Fredholm integral equations, modified decomposition, restarted Adomian method**.**

1. Introduction

System of integral equations are used as mathematical models for many physical situations, and also occur as reformulations of other mathematical problems. Recently a great deal of interest has been focused on application of the Adomian decomposition method to solve a wide variety of problems [1, 2, 21, 24]. The Adomian decomposition method has been applied for solving system of linear Fredholm integral equations of the second kind [7, 9]. Also, many different method have been used to approximate the solution of linear Fredholm integral equations system of the second kind, for example Taylor-series expansion method [18], Chebyshev collocation method [4, 12] and block-pulse functions method[17]. Recently, many different method have been used to estimate the solution of the system of nonlinear Fredholm integral equations, fox example Taw method [16], homotopy method [19, 20] and hat basis functions method [8]. In [10], an alternate algorithm for computing Adomian polynomials was present. That method gave better approximations with less iteration when applied to solve both linear and nonlinear system of Volterra integral equations [5, 6]. The reason could be found in the point that the system of Volterra integral equations of the second kind is basically more well posed than the system of Fredholm integral equations of the second kind [14]. Accelerating the convergence of the Adomian decomposition method when applied to a system of Fredholm integral equations of the second kind is a good subject for further research [10].

In this paper, we use three modifications of Adomian decomposition method for solving system of nonlinear Fredholm integral equations. The first one is the modified decomposition method [23]. The author in [23] applied the method to solve a system of nonlinear Volterra integral equations. The second one is the new modification of Adomian decomposition method [22]. In [22], the authors applied their method to several modeling nonlinear problems. The third one is the restarted Adomian method [15]. The method is applied to solve system of nonlinear Volterra integral equations [15].

Now, we consider the system of Fredholm equations

$$F(t) = G(t) + \int_a^b V(s,t,F(s))\, ds, t \in [a,b] \qquad (1)$$

where

$V(s,t,F(s)) = (v_1(s,t,F(s)), v_2(s,t,F(s)),...,v_n(s,t,F(s)))^T$,

$F(t) = (f_1(t), f_2(t),..., f_n(t))^T$ and $G(t) = (g_1(t), g_2(t),..., g_n(t))^T$. We suppose that, the system (1) has a unique solution. However, the necessary and sufficient conditions for existence and uniqueness of solution could be found in [13].

2. The Adomian Decomposition Method

Consider the $i^{\underline{th}}$ equation of (1)

$$f_i(t) = g_i(t) + \int_a^b v_i(s,t, f_1(s), f_2(s),..., f_n(s))\, ds \qquad (2)$$

The canonical form of the Adomian equations can be written as

$$f_i(t) = g_i(t) + N_i(t) \qquad (3)$$

where

$$N_i(t) = N_i(f_1, f_2,..., f_n)(t) = \int_a^b v_i(s,t, f_1(s), f_2(s),..., f_n(s))\, ds \qquad (4)$$

To use the Adomian decomposition method, we put $f_i(t) = \sum_{m=0}^{\infty} f_{im}(t)$ and $N_i(t) = \sum_{m=0}^{\infty} A_{im}$, where $A_{im}, m = 0,1,2,\dots$, are the Adomian polynomials. Hence (3) can be written as

$$\sum_{m=0}^{\infty} f_{im}(t) = g_i(t) + \sum_{m=0}^{\infty} A_{im}(f_{10}, f_{11}, \dots, f_{1m}, \dots, f_{n0}, \dots, f_{nm}) \qquad (5)$$

From (5), we define

$$f_{i0}(t) = g_i(t)$$

$$f_{i,m+1}(t) = A_{im}(f_{10}, \dots, f_{1m}, \dots, f_{n0}, \dots, f_{nm}),\ i = 1,2,\dots,n\ ,\ m = 0,1,2,\dots (6)$$

In practice, all terms of the series $f_i(t) = \sum_{m=0}^{\infty} f_{im}(t)$ cannot be determined and so we have an approximation of the solution by the following truncated series $\varphi_{ik}(t) = \sum_{m=0}^{k-1} f_{im}(t)$ with

$$\lim_{k\to\infty} \varphi_{ik}(t) = f_i \qquad (7)$$

To determine the Adomian polynomials, we write

$$f_{i\lambda}(t) = \sum_{m=0}^{\infty} f_{im}(t) \quad \lambda^m \qquad (8)$$

and

$$N_{i\lambda}(f_1, f_2, \dots, f_n) = \sum_{m=0}^{\infty} A_{im} \quad \lambda^m, \qquad (9)$$

where λ is a parameter introduced for convenience. From (9), we obtain

$$A_{im}(t) = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m} N_{i\lambda}(f_1, f_2, \dots, f_n)\right]_{\lambda=0} \qquad (10)$$

3. The Convergence

We consider the system of nonlinear Fredholm integral equations (1), and let us consider the i<u>th</u> equation of (1), and using Adomian decomposition method to get equation (3), where N_i is the nonlinear operator. Let

$$\rho = \min\{\ \rho_1, \rho_2, \dots, \rho_n\ \} \qquad (11)$$

where ρ_i is the convergence radius of series(8), and $\rho_i > 1$, thus (8)converges for $|\lambda| \le \rho$ with $\rho > 1$. Now suppose that $N_i(f_1, f_2, \dots, f_n)$ can be expanded in an entire series [5]

$$N_i(f_1, f_2, \dots, f_n) = \sum_{m=0}^{\infty} \sum_{\substack{k_1+\dots+k_m=m \\ k_1,\dots,k_m \in W\{0,1,\dots\}}} a_{k_1 k_2 \dots k_m} \quad f_1^{k_1}\ f_2^{k_2} \dots\ f_m^{k_m} \qquad (12)$$

3.1 Theorem

If $N(\overline{F})$ is an analytic function of n variables $f_1, f_2, \dots, f_n$ in $\overline{F} < R$, $f_i(t), i = 1,2,\dots,n$ which can be decomposed into infinite series $f_i = \sum f_{im}$, the parametrization $f_{i\lambda} = \sum f_{im}\lambda^m$ is absolutely convergent for $\lambda \in [-1,1]$ and the series f_i can be majored by

$$\frac{u'}{n(1+\varepsilon)}\left[1+\frac{1}{1+\varepsilon}\frac{\lambda}{\rho}+\ \dots\ +\frac{1}{(1+\varepsilon)^n}\left(\frac{\lambda}{\rho}\right)^n\right] \quad (13)$$ where

$$u' \geq u \quad , \quad u = \max\{u_1, u_2, \ \dots\ , u_n\}$$

u_i is the upper limit of the series, element $\varepsilon > \frac{u}{R}$, and $\rho \geq 1$, then the double series converges for $\lambda = 1$.

4. The Modifications of the Adomian Decomposition Method

(i) The modified Decomposition Method

The assumptions made by Adomian [3] were modified by Wazwaz [23], the modified form was established based on the assumption that the function g_i can be divided into two parts, namely g_{0i} and g_{1i}. Under this assumption we set

$$g_i(t) = g_{0i}(t) + g_{1i}(t) \quad (14)$$

Accordingly, a slight variation was proposed only on the components f_{0i} and f_{1i}. The suggestion was that only the part g_{0i} be assigned to the zeroth component f_{0i}, whereas the remaining part g_{1i} be combined with the other terms to difine f_{1i}. Consequently the modified recursive relation

$$\begin{aligned} f_{0i}(t) &= g_{0i}(t) \\ f_{1i}(t) &= g_{1i}(t) - L^{-1}(A_{0i}) \\ f_{k+2,i}(t) &= -L^{-1}(A_{k+1,i}) \quad , \quad k \geq 0 \end{aligned} \quad (15)$$

The choice of g_{0i} such that f_{0i} contains the minimal number of terms has a strong influence accelerates the convergence of the solution. Thus means that the success of this method depends mainly on the proper choice of g_{0i} and g_{1i}.We have been unable to establish any criteria to judge what forms of g_{0i} and g_{1i} can be used to yield the acceleration demanded, at present g_{0i} and g_{1i} are selected by trials. The modification demonstrate a rapid convergence of the series solution if compared with the standard Adomian decomposition method, and it may give the exact solution for nonlinear equations by using two iterations only and without using the so-called Adomian polynomials.

(ii) A New Modification of Adomian decomposition method

The modified decomposition method in **(i)** depends entirely on the proper selection of the functions g_{0i} and g_{1i}. It appears that trials are the only criteria that can be applied so far. In the new modification by Wazwaz [22] we can replace the process of dividing g_i into two components by a series of infinite components. We therefore suggest that g_i be expressed in Taylor series

$$g_i(t) = \sum_{n=0}^{\infty} g_{in}(t) \qquad (16)$$

A new recursive relationship expressed in the form

$$f_{0i}(t) = g_{0i}(t)$$

$$f_{i,k+1}(t) = g_{i,k+1}(t) - L^{-1}(A_{ik}) \qquad ,k \geq 0 \qquad (17)$$

It is important to note that if g_i consists of one term only, then scheme (17) reduces to relation(6). Moreover if g_i consists of two terms, then relation(17)reduces to the modified relation(15).

If the computation of $N_i(f_1, f_2,..., f_n)$ in equation(10) is very complicated we can consider the Taylor expansion of $N_i(f_1, f_2,..., f_n)$ and consider a few first terms of the expansion and then apply the main idea of the Adomian algorithm [11].

We can observe that algorithm (17) reduces the number of terms involved in each component, and hence the size of calculations is minimized compared to the standard Adomian decomposition method only. Moreover this reduction of terms in each component facilitates the construction of Adomian polynomials for nonlinear operators. The new modification overcomes the difficulty of decomposing $f(x)$, and introduces an efficient algorithm that improves the performance of the standard Adomian decomposition method.

(iii) Restarted Adomian Method

The restarted Adomian method was used in [15] as a new method based on standard Adomian method for solving system of nonlinear Volterra integral equations. In this method we use the modified Adomian method which proposed a slight variation only on the components u_0 and u_1, and restarted Adomian method applied to algebraic equations. The rate of the method is more accelerate than standard Adomian method. We introduce the algorithm as the following

1-Choose small natural numbers m, k.

2-Apply the Adomian method on equation (3) and calculate

$$f_{0i}, f_{1i}, f_{2i},..., f_{ki}, i = 1,2,3,..., n ,$$

Set

$$\varphi_i^1 = f_{0i} + f_{1i} + f_{2i} + ... + f_{ki} \qquad (18)$$

3-Let z_i be the proper function which will be determined next, for $j = 2 : m$

$$[z_1 \quad z_2 \quad z_3 \quad \ldots \quad z_n]^t = [\varphi_1^{j-1} \quad \varphi_2^{j-1} \quad \ldots \quad \varphi_n^{j-1}]^t \tag{19}$$

$$[f_{01} \quad f_{02} \quad f_{03} \quad \ldots \quad f_{0n}]^t = [z_1 \quad z_2 \quad \ldots \quad z_n]^t \tag{20}$$

$$\begin{bmatrix} f_{11} \\ f_{12} \\ . \\ . \\ . \\ f_{1n} \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ . \\ . \\ . \\ g_n \end{bmatrix} - \begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ . \\ z_n \end{bmatrix} + \begin{bmatrix} A_{01}(f_{01}, f_{02}, \ldots, f_{0n}) \\ A_{02}(f_{01}, f_{02}, \ldots, f_{0n}) \\ . \\ . \\ . \\ A_{0n}(f_{01}, f_{02}, \ldots, f_{0n}) \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} f_{21} \\ f_{22} \\ . \\ . \\ . \\ f_{2n} \end{bmatrix} = \begin{bmatrix} A_{11}(f_{01}, f_{02}, \ldots, f_{0n}) \\ A_{12}(f_{01}, f_{02}, \ldots, f_{0n}) \\ . \\ . \\ . \\ A_{1n}(f_{01}, f_{02}, \ldots, f_{0n}) \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} f_{k+1,1} \\ f_{k+1,2} \\ . \\ . \\ . \\ f_{k+1,n} \end{bmatrix} = \begin{bmatrix} A_{k1}(f_{01}, f_{11}, \ldots, f_{k1}, \ldots, f_{0n}, f_{1n}, \ldots, f_{kn}) \\ A_{k2}(f_{01}, f_{11}, \ldots, f_{k1}, \ldots, f_{0n}, f_{1n}, \ldots, f_{kn}) \\ . \\ . \\ . \\ A_{kn}(f_{01}, f_{11}, \ldots, f_{k1}, \ldots, f_{0n}, f_{1n}, \ldots, f_{kn}) \end{bmatrix} \tag{23}$$

4- Set

$$\varphi_i^j = f_{0i} + f_{1i} + f_{2i} + \ldots + f_{ki} \qquad , i = 1,2,\ldots, n \tag{24}$$

Remarks:

- $(\varphi_1^m, \varphi_2^m, \ldots, \varphi_n^m)^T$ can be considered as an approximation of solution of (1).
- The Adomian method usually gives sum of the some first terms as an approximation of $F = (f_1, f_2, \ldots, f_n)^T$ in this algorithm we can update $f_{0i}, i = 1,2,3,\ldots, n$ in each step, but we don't calculate the terms with large index, so m and k are considered small.

5. Numerical Results

Example(1) :

Consider the following system of nonlinear Fredholm integral equations of the second kind [20] with the exact solutions $f_1(t) = t \quad and \quad f_2(t) = t^2$.

$$f_1(t) = t - \frac{1}{4}e^{-t} + \int_0^1 e^{-t} f_1(s) f_2(s)\, ds$$
$$f_2(t) = t^2 - \frac{1}{5}e^{-t} + \int_0^1 e^{-t} f_1^2(s) f_2(s)\, ds \qquad (25)$$

Adomian decomposition method for this problem consists of the following scheme

$$f_{10}(t) = t - \frac{1}{4}e^{-t}$$
$$f_{20}(t) = t^2 - \frac{1}{5}e^{-t} \qquad (26)$$
$$f_{1,m+1}(t) = A_{1m}(f_{10},\dots, f_{1m}, f_{20},\dots, f_{2m})$$
$$f_{2,m+1}(t) = A_{2m}(f_{10},\dots, f_{1m}, f_{20},\dots, f_{2m}), \quad m = 0,1,2,\dots \qquad (27)$$

For the first iteration, we have

$$f_{11}(t) = A_{10}(f_{10}, f_{20}) = 0.1786176958 \quad e^{-t}$$
$$f_{21}(t) = A_{20}(f_{10}, f_{20}) = 0.1268575374 \quad e^{-t}$$

and for the second iteration, we have

$$f_{12}(t) = A_{11}(f_{10}, f_{11}, f_{20}, f_{21}) = \int_0^1 e^{-t}(f_{10} \; f_{21} + f_{20}\, f_{11})\, ds$$
$$f_{22}(t) = A_{21}(f_{10}, f_{11}, f_{20}, f_{21}) = \int_0^1 e^{-t}(f_{10}^{\;2} \; f_{21} + 2\, f_{10}\, f_{20}\, f_{11})\, ds$$

Similarly

$$f_{12}(t) = 0.0347784 \quad e^{-t} \quad , \quad f_{22}(t) = 0.0407118 \quad e^{-t} \qquad (29)$$

The solutions with three terms are

$$\varphi_{1,3} = f_{10} + f_{11} + f_{12} = t - 0.0366039 \quad e^{-t}$$
$$\varphi_{2,3} = f_{20} + f_{21} + f_{22} = t^2 - 0.0324307 \quad e^{-t} \qquad (30)$$

The solutions after four iteration and for the first five terms are give as

$$\varphi_{1,5} = t - 0.0147857035 \quad e^{-t} \quad , \varphi_{2,5} = t^2 - 0.009912119 \quad e^{-t} \qquad (31)$$

(i) The modified Decomposition Method

The following modified scheme can be used to solve system(25)

$$f_{10}(t) = t \quad , \quad f_{20}(t) = t^2$$
$$f_{11}(t) = -\frac{1}{4}e^{-t} + \int_0^1 e^{-t} f_1(s) f_2(s)\, ds = 0$$
$$f_{21}(t) = -\frac{1}{5}e^{-t} + \int_0^1 e^{-t} f_1^2(s) f_2(s)\, ds = 0$$

So the exact solution is $f_1(t) = t$ *and* $f_2(t) = t^2$.

(ii) A New Modification

We first set the Taylor expansion for $g_i(t)$

$$g_{1i}(t) = t - \frac{1}{4}e^{-t} \cong t - \frac{1}{4} + \frac{1}{4}t - \frac{1}{8}t^2 + \frac{1}{24}t^3 - \frac{1}{96}t^4 + \frac{1}{480}t^5 + O[t]^6$$
$$g_{2i}(t) = t^2 - \frac{1}{5}e^{-t} \cong t^2 - \frac{1}{5} + \frac{1}{5}t - \frac{1}{10}t^2 + \frac{1}{30}t^3 - \frac{1}{120}t^4 + \frac{1}{600}t^5 + O[t]^6 \qquad (32)$$

Following relation (17) we obtain

$$f_{10}(t) = t \quad , \quad f_{20}(t) = t^2$$

Then

$$f_{11}(t) = \frac{-1}{4} + \int_0^1 e^{-t}(s)(s^2)ds = -0.25 + 0.25e^{-t}$$

$$f_{21}(t) = \frac{-1}{5} + \int_0^1 e^{-t}(s)^2(s^2)ds = -0.2 + 0.2e^{-t}$$

$$f_{12}(t) = \frac{1}{4}t + A_{11}(f_{10}, f_{11}, f_{20}, f_{21}) = \frac{1}{4}t + \int_0^1 e^{-t}(f_{10} \quad f_{21} + f_{20}f_{11})\, ds$$
$$= -0.0903344112 \quad e^{-t} + 0.25t$$

$$f_{22}(t) = \frac{1}{5}t + A_{21}(f_{10}, f_{11}, f_{20}, f_{21}) = \frac{1}{5}t + \int_0^1 e^{-t}(f_{10}{}^2 \quad f_{21} + 2f_{10}f_{20}f_{11})\, ds$$
$$= -0.102581637 \quad e^{-t} + 0.2t$$

$$f_{13}(t) = \frac{-1}{8}t^2 + \tfrac{1}{2}\int_0^1 e^{-t}(2f_{10} \quad f_{22} + 2f_{11}f_{21} + 2f_{20}f_{12})\, ds$$
$$= 0.09595698 \quad e^{-t} - 0.125t^2$$

$$f_{23}(t) = \frac{-1}{10}t^2 + \tfrac{1}{2}\int_0^1 e^{-t}(2f_{10}{}^2 \quad f_{22} + 4f_{10}f_{11}f_{21} + f_{20}(2f_{11}{}^2 + 4f_{10}f_{12}))\, ds$$
$$= 0.1307532400 \quad 9e^{-t} - 0.1t^2$$

The solutions with three terms are

$$f_1 = f_{10} + f_{11} + f_{12} = -0.25 + 0.2556225687 \quad e^{-t} + 1.25t - 0.125t^2$$
$$f_2 = f_{20} + f_{21} + f_{22} = -0.2 + 0.2281716030 \quad 89e^{-t} + 0.2t + 0.9t^2 \qquad (33)$$

(ii) Restarted Adomian Method

Consider small indexes m *and* k, say $m = 2$, $k = 2$

Step1: The first two terms and corresponding partial sums are given the following table

i	f_{1i}	f_{2i}
0	t	t^2
1	0	0

Table 1

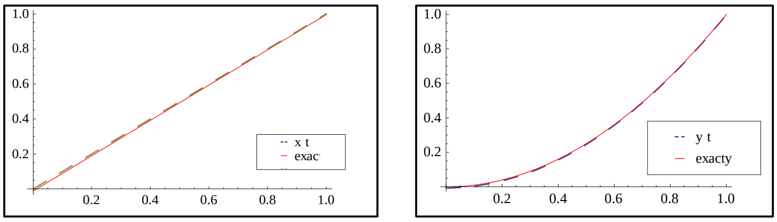
$$\varphi_1^1 = f_{10} + f_{11} = t \quad , \quad \varphi_2^1 = f_{20} + f_{21} = t^2$$

Step2: let $f_{10} = \varphi_1^1 \quad , \quad f_{20} = \varphi_2^1$

Applying relation (28) to get $f_{11} = 0 \quad , \quad f_{21} = 0$

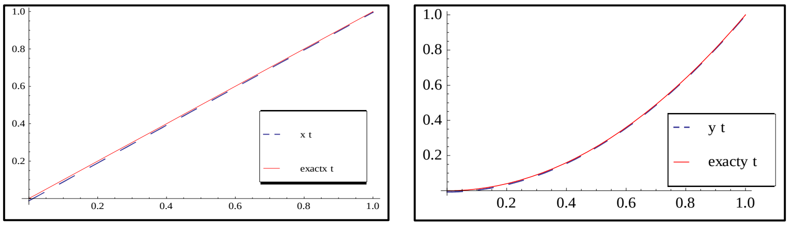
So the exact solution is $f_1(t) = t$ *and* $f_2(t) = t^2$.

The efficient of each modification methods after 10 iterations, compare to exact solutions, present in Table2 and 3.



(a) (b)

Fig.1. comparison of the exact solution with Adomian decomposition method for Ex.1 , (a) for f_1 and (b) for f_2 .



(a) (b)

Fig.2. comparison of the exact solution with New Modification of decomposition method for Ex.1 , (a) for f_1 and (b) for f_2 .

Example(2) :

Consider the following system of nonlinear Fredholm integral equations of the second kind [19] with the exact solutions $f_1(t) = x - 1$ *and* $f_2(t) = t^2$.

$$f_1(t) = g_1(t) + \int_0^1 (s\,t\, f_1^2(s) + s^2\, f_2^3(s))\,ds$$

$$f_2(t) = g_2(t) + \int_0^1 (s\, f_1^3(s) - t\, f_2(s))^2\, ds$$

Where $g_1(t) = \frac{-10}{9} + \frac{11}{12}t \;,\; g_2(t) = \frac{-1}{252} - \frac{1}{70}t + \frac{4}{5}t^2$

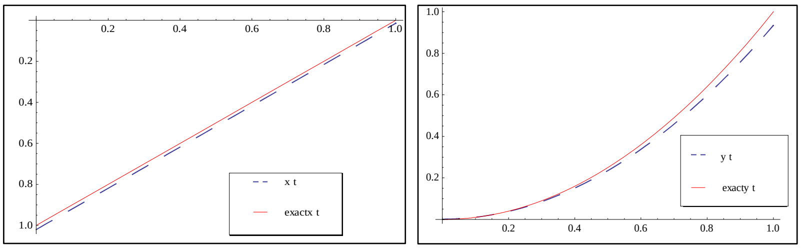
Adomian decomposition method for this problem

$$f_{10}(t) = g_1 \;,\; f_{20}(t) = g_2$$

The solutions with 10 terms are

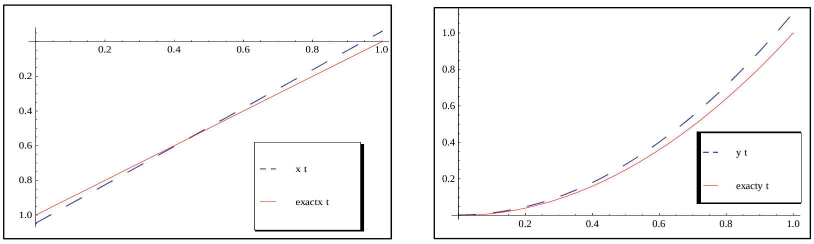
$$f_1(t) = 1.00827 \quad t - 1.01987$$
$$f_2(t) = 0 - 934374 \quad t^2 + 0.00398802 \quad t + 0.0131053$$

The efficient of standard and modified decompositiom methods after 10 iterations, compare to exact solutions, present in Table4 and 5.



(a) (b)

Fig.3. comparison of the exact solution with Adomian decomposition method for Ex.2 , (a) for f_1 and (b) for f_2 .



(a) (b)

Fig.4.comparison of the exact solution with Modified decomposition method for Ex.2,(a) for f_1 and (b) for f_2 .

6. Conclusion

t	f_{1i}				
	exact	**standard**	$error_s$	**New modification**	$error_n$
0.1	**0.1**	**0.0890423**	1.21102×10^{-2}	**0.089082**	1.20653×10^{-2}
0.2	**0.2**	**0.190085**	1.0957×10^{-2}	**0.190122**	1.09171×10^{-2}
0.3	**0.3**	**0.291029**	$9,91496 \times 10^{-3}$	**0.291062**	9.8782×10^{-3}
0.4	**0.4**	**0.392882**	8.971496×10^{-3}	**0.391912**	8.93816×10^{-3}
0.5	**0.5**	**0.491882**	8.11768×10^{-3}	**0.492682**	7.31795×10^{-3}
0.6	**0.6**	**0.593354**	7.34518×10^{-3}	**0.593378**	6.62155×10^{-3}
0.7	**0.7**	**0.693354**	6.6462×10^{-3}	**0.694009**	5.99143×10^{-3}
0.8	**0.8**	**0.794559**	6.01373×10^{-3}	**0.794579**	5.42126×10^{-3}
0.9	**0.9**	**0.895076**	5.4414×10^{-3}	**0.895095**	4.90535×10^{-3}
1.0	**1.0**	**0.995545**	4.45508×10^{-3}	**0.995562**	4.4385×10^{-3}

Table 2

t	f_{2i}				
	exact	**standard**	$error_s$	**New modification**	$error_n$
0.1	**0.01**	**0.003691**	6.25324×10^{-3}	**0.00381246**	6.18754×10^{-3}
0.2	**0.04**	**0.037467**	5.65817×10^{-3}	**0.0344013**	5.59871×10^{-3}
0.3	**0.09**	**0.084880**	5.11972×10^{-3}	**0.0849341**	5.06593×10^{-3}
0.4	**0.16**	**0.155367**	4.63252×10^{-3}	**0.155416**	4.58384×10^{-3}
0.5	**0.25**	**0.245808**	4.19167×10^{-3}	**0.245852**	4.14763×10^{-3}
0.6	**0.36**	**0.356207**	3.79278×10^{-3}	**0.356247**	3.75293×10^{-3}
0.7	**0.49**	**0.486568**	3.43185×10^{-3}	**0.486604**	3.395797×10^{-3}
0.8	**0.64**	**0.636895**	3.10527×10^{-3}	**0.636927**	3.07263×10^{-3}
0.9	**0.81**	**0.807190**	2.80976×10^{-3}	**0.80722**	2.78022×10^{-3}
1.0	**1.0**	**0.997458**	2.54238×10^{-3}	**0.997484**	2.51561×10^{-3}

Table 3

t	f_{2i}						
	exact	**standard**	$error_s$	**Modified decomposition**	$error_m$	**Restart Method**	$error_r$
0.1	**0.01**	**0.0228478**	1.28478×10^{-2}	**0.0108145**	8.14462×10^{-3}	**0.013371**	2.04164×10^{-3}
0.2	**0.04**	**0.0512779**	1.12779×10^{-2}	**0.0389342**	1.06578×10^{-3}	**0.0468332**	3.37104×10^{-3}
0.3	**0.09**	**0.0983954**	8.39538×10^{-3}	**0.0856841**	4.31593×10^{-3}	**0.102428**	6.83315×10^{-3}
0.4	**0.16**	**0.1642**	4.2004×10^{-3}	**0.0108145**	8.9359×10^{-3}	**0.180156**	1.2428×10^{-2}
0.5	**0.25**	**0.248693**	1.3071×10^{-3}	**0.235074**	1.49259×10^{-3}	**0.280016**	2.01555×10^{-2}
0.6	**0.36**	**0.351873**	8.12711×10^{-3}	**0.337714**	3.10155×10^{-2}	**0.402009**	3.00157×10^{-2}
0.7	**0.49**	**0.47374**	1.62596×10^{-2}	**0.458985**	1.06578×10^{-2}	**0.546134**	4.20087×10^{-2}
0.8	**0.64**	**0.614295**	2.57047×10^{-2}	**0.598885**	4.11151×10^{-2}	**0.712393**	4.11151×10^{-2}
0.9	**0.81**	**0.773538**	3.64622×10^{-2}	**0.757415**	5.25846×10^{-2}	**0.900784**	7.23927×10^{-2}
1.0	**1.0**	**0.951468**	4.85323×10^{-2}	**0.934576**	6.54248×10^{-2}	**1.11131**	9.07838×10^{-2}

Table 4

t	f_{1i}						
	exact	**standard**	$error_s$	**Modified decomposition**	$error_n$	**Restart Method**	$error_r$
0.1	**-0.9**	**-0.919044**	1.90442×10^{-2}	**-0.919649**	1.96486×10^{-2}	**-0.935849**	4.64072×10^{-2}
0.2	**-0.8**	**-0.818217**	1.82174×10^{-2}	**-0.819059**	1.90588×10^{-2}	**-0.82529**	3.58487×10^{-2}
0.3	**-0.7**	**-0.717391**	1.73905×10^{-2}	**-0.718469**	1.84691×10^{-2}	**-0.71473**	2.52902×10^{-2}
0.4	**-0.6**	**-0.616564**	1.65637×10^{-2}	**-0.617879**	1.78794×10^{-2}	**-0.604173**	1.47316×10^{-2}
0.5	**-0.5**	**-0.515737**	1.57369×10^{-2}	**-0.517879**	1.72896×10^{-2}	**-0.493615**	4.17312×10^{-3}
0.6	**-0.4**	**-0.41491**	1.491×10^{-2}	**-0.4167**	1.66999×10^{-2}	**-0.383056**	6.38541×10^{-3}
0.7	**-0.3**	**-0.314083**	1.40832×10^{-2}	**-0.31611**	1.61101×10^{-2}	**-0.272498**	1.69439×10^{-2}
0.8	**-0.2**	**-0.213256**	1.32563×10^{-2}	**-0.21552**	1.55204×10^{-2}	**-0.161939**	2.75024×10^{-2}
0.9	**-0.1**	**-0.112429**	1.24295×10^{-2}	**-0.114931**	1.49307×10^{-2}	**-0.05138**	3.8061×10^{-2}
1.0	**0**	**-0.011602**	1.16027×10^{-2}	**-0.0143409**	1.43409×10^{-2}	**0.059178**	4.64072×10^{-2}

Table 5

We applied some modifications of Adomian decomposition method for solving system of nonlinear Fredholm integral equations of the second kind. A comparative between the modifications method and the standard decomposition method is present from some examples to show the efficiency of each method. The modified method is useful for acquiring the solution as demonstrated in examples, the exact solution is obtained by using two iterations only for all examined models . A necessary condition for that is, if the exact solution must be a part of the component of the zeroth component.

The only necessary and sufficient assumption for applying improvement method is that the existence of the Taylor expansion of $g_i(t)$ but this assumption is guaranteed already. Since in the canonical form of the equation there is mentioned that $g_i(t)$ is analytical. The standard Adomian method gives better approximate than the restarted Adomian method. The computations associated with the examples discussed above were preformed by using Mathematica 7.

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