CHAPTER 20

The Second Law of Thermodynamics

1* · Where does the energy come from in an internal-combustion engine? In a steam engine?

Internal combustion engine: From the heat of combustion (see Problems 19-106 to 19-109).

Steam engine: From the burning of fuel to evaporate water and to raise the temperature and pressure of the steam.

2 · How does friction in an engine affect its efficiency?

Friction reduces the efficiency of the engine.

3 · John is house-sitting for a friend who keeps delicate plants in her kitchen. She warns John not to let the room get too warm or the plants will wilt, but John forgets and leaves the oven on all day after his brownies are baked. As the plants begin to droop, John turns off the oven and opens the refrigerator door, intending to use the refrigerator to cool the kitchen. Explain why this doesn't work.

Since a refrigerator exhausts more heat to the room than it extracts from the interior of the refrigerator, the temperature of the room will increase rather than decrease.

4 · Why do power-plant designers try to increase the temperature of the steam fed to engines as much as possible?

Increasing the temperature of the steam increases the Carnot efficiency, and generally increases the efficiency of any heat engine.

5* · An engine with 20% efficiency does 100 J of work in each cycle. (*a*) How much heat is absorbed in each cycle? (*b*) How much heat is rejected in each cycle?

(*a*) From Equ. 20-2, $Q_h = W/\varepsilon$ $\quad$ $Q_h = 100/0.2\text{ J} = 500\text{ J}$

(*b*) $|Q_c| = Q_h(1 - \varepsilon)$ $\quad$ $|Q_c| = 500 \times 0.8\text{ J} = 400\text{ J}$

6 · An engine absorbs 400 J of heat and does 120 J of work in each cycle. (*a*) What is its efficiency? (*b*) How much heat is rejected in each cycle?

(*a*) $\varepsilon = W/Q_h$ $\quad$ $\varepsilon = 120/400 = 0.3 = 30\%$

(*b*) $|Q_c| = Q_h(1 - \varepsilon)$ $\quad$ $|Q_c| = 400 \times 0.7\text{ J} = 280\text{ J}$

7 · An engine absorbs 100 J and rejects 60 J in each cycle. (*a*) What is its efficiency? (*b*) If each cycle takes

0.5 s, find the power output of this engine in watts.

(*a*) Use Equ. 20-2 — $e = 1 - 60/100 = 0.4 = 40\%$

(*b*) $P = W/\Delta t = e\, Q_h/\Delta t$ — $P = 0.4 \times 100/0.5$ W = 80 W

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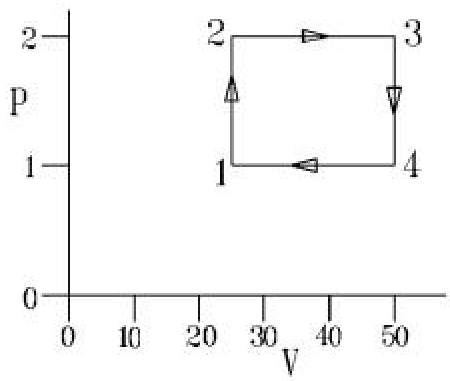
8 · A refrigerator absorbs 5 kJ of energy from a cold reservoir and rejects 8 kJ to a hot reservoir. (*a*) Find the coefficient of performance of the refrigerator. (*b*) The refrigerator is reversible and is run backward as a heat engine between the same two reservoirs. What is its efficiency?

(*a*) $W = |Q_h| - Q_c$; COP = Q_c/W — $W = 3$ kJ; COP = 5/3 = 1.67

(*b*) $e = W/Q_h$ — $e = 3/8 = 0.375 = 37.5\%$

9* ·· An engine operates with 1 mol of an ideal gas for which $C_v = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$ as its working substance. The cycle begins at $P_1 = 1$ atm and $V_1 = 24.6$ L. The gas is heated at constant volume to $P_2 = 2$ atm. It then expands at constant pressure until $V_2 = 49.2$ L. During these two steps, heat is absorbed by the gas. The gas is then cooled at constant volume until its pressure is again 1 atm. It is then compressed at constant pressure to its original state. During the last two steps, heat is rejected by the gas. All the steps are quasi-static and reversible. (*a*) Show this cycle on a *PV* diagram. Find the work done, the heat added, and the change in the internal energy of the gas for each step of the cycle. (*b*) Find the efficiency of the cycle.

(*a*) The cycle is shown on the right. Here, the pressure *P* is in atm and the volume *V* is in L. To determine the heat added during each step shall first find the temperatures at points 1, 2, 3, and 4.



$T_1 = 24.6 \times 273/22.4$ K = 300 K

$T_2 = 2T_1 = 600$ K

$T_3 = 2T_2 = 1200$ K

$T_4 = 2T_1 = 600$ K

$W_{1\text{-}2} = P\Delta V_{1\text{-}2}$; $Q_{1\text{-}2} = \Delta U_{12} = C_v\Delta T_{1\text{-}2}$ — $W_{1\text{-}2} = 0$; $Q_{1\text{-}2} = 1.5 \times 8.314 \times 300$ J = 3.74 kJ = $\Delta U_{1\text{-}2}$

$W_{2\text{-}3} = P\Delta V_{2\text{-}3}$; $Q_{2\text{-}3} = C_p\Delta T_{2\text{-}3}$ — $W_{2\text{-}3} = 2 \times 24.6$ atm·L = 4.97 kJ; $Q_{2\text{-}3} = 2.5 \times 8.314 \times 600$ J = 12.47 kJ; $\Delta U_{2\text{-}3} = (12.47 - 4.97)$ kJ = 7.5 kJ

$W_{3\text{-}4} = P\Delta V_{3\text{-}4}$; $Q_{3\text{-}4} = C_v\Delta T_{3\text{-}4}$ — $W_{3\text{-}4} = 0$; $Q_{34} = -1.5 \times 8.314 \times 600$ J = -7.48 kJ = $\Delta U_{3\text{-}4}$

$W_{4\text{-}1} = P\Delta V_{4\text{-}1}$; $Q_{4\text{-}1} = C_p\Delta T_{4\text{-}1}$ — $W_{4\text{-}1} = -24.6$ atm·L = -2.48 kJ; $Q_{4\text{-}1} = -6.24$ kJ; $\Delta U_{4\text{-}1} = -3.76$ kJ

(*b*) $e = W/Q_{in}$ — $W = 2.48$ kJ; $Q_{in} = 16.21$ kJ. $e = 0.153 = 15.3\%$

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10 ·· An engine using 1 mol of a diatomic ideal gas performs a cycle consisting of three steps: (1) an adiabatic expansion from an initial pressure of 2.64 atm and an initial volume of 10 L to a pressure of 1 atm and a volume of 20 L, (2) a compression at constant pressure to its original volume of 10 L, and (3) heating at constant volume to its original pressure of 2.64 atm. Find the efficiency of this cycle.

The first process is adiabatic.	$Q_1 = 0$
Second step: $Q_2 = C_p\Delta T_2 = 3.5P\Delta V_2$	$Q_2 = -35$ atm·L
Third step: $Q_3 = C_v\Delta T_3 = 2.5V_3\Delta P_3$	$Q_3 = 25 \times 1.64$ atm·L = 41 atm·L
For cycle, $\Delta U = 0$, so $W = Q_1 + Q_2 + Q_3$	$W = 6$ atm·L
$e = W/Q_{in}$	$e = 6/41 = 0.146 = 14.6\%$

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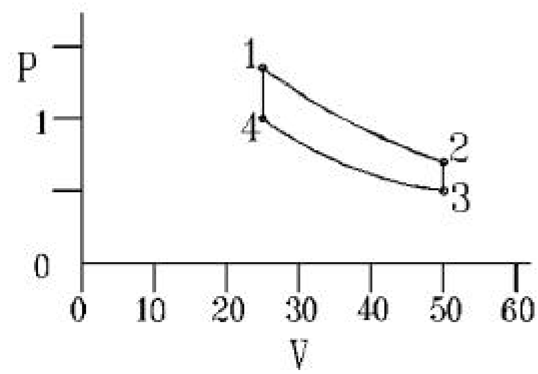
11 ·· An engine using 1 mol of an ideal gas initially at $V_1 = 24.6$ L and $T = 400$ K performs a cycle consisting of four steps: (1) an isothermal expansion at $T = 400$ K to twice its initial volume, (2) cooling at constant volume to $T = 300$ K, (3) an isothermal compression to its original volume, and (4) heating at constant volume to its original temperature of 400 K. Assume that $C_v = 21$ J/K. Sketch the cycle on a *PV* diagram and find its efficiency.

1. The cycle is shown in the figure. Here *P* is in atm and *V* is in L.
2. From the data, $T_1 = T_2 = 400$ K, $T_3 = T_4 = 300$ K, $V_1 = V_4 = 24.6$ L, and $V_2 = V_3 = 49.2$ L.
3. We next determine the work done and the heat absorbed by the gas during each of the four steps.



Step 1-2: $W_{1\text{-}2} = nRT_1 \ln(V_2/V_1) = Q_{1\text{-}2}$	$W_{1\text{-}2} = 8.314 \times 400 \times \ln(2)$ J = 2.305 kJ = $Q_{1\text{-}2}$
Step 2-3: $W_{2\text{-}3} = P\Delta V$; $Q_{23} = C_v\Delta T$	$W_{2\text{-}3} = 0$; $Q_{2\text{-}3} = -21 \times 100$ J = -2.1 kJ
Step 3-4: $W_{3\text{-}4} = nRT_3 \ln(V_4/V_3) = Q_{3\text{-}4}$	$W_{3\text{-}4} = 8.314 \times 300 \times \ln(0.5)$ J = -1.729 kJ = $Q_{3\text{-}4}$
Step 4-1: $W_{4\text{-}1} = P\Delta V$; $Q_{41} = C_v\Delta T$	$W_{4\text{-}1} = 0$; $Q_{4\text{-}1} = 21 \times 100$ J = 2.1 kJ
Find W and Q_{in}	$W = (2.305 - 1.729)$ kJ = 0.576 kJ; $Q_{in} = 4.405$ kJ
Determine $e = W/Q_{in}$	$e = 0.576/4.405 = 0.131 = 13.1\%$

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12 ·· One mole of an ideal monatomic gas at an initial volume $V_1 = 25$ L follows the cycle shown in Figure 20-11. All the processes are quasi-static. Find (*a*) the temperature of each state of the cycle, (*b*) the heat flow for each part of the cycle, and (*c*) the efficiency of the cycle.

(*a*) Use $PV = nRT$	$T_1 = 100 \times 25/8.314$ K = 300.7 K; $T_2 = T_3 = 601.4$ K
(*b*) $Q_{1\text{-}2} = C_v\Delta T$	$Q_{1\text{-}2} = 1.5 \times 8.314 \times 300.7$ J = 3.75 kJ
$Q_{2\text{-}3} = W_{2\text{-}3} = nRT_2 \ln(V_3/V_2)$	$Q_{2\text{-}3} = 8.314 \times 601.4 \times \ln(2)$ J = 3.466 kJ
$Q_{3\text{-}1} = C_p\Delta T$	$Q_{3\text{-}1} = -2.5 \times 8.314 \times 300.7$ J = -6.25 kJ
(*c*) $W = \Sigma Q$; $Q_{in} = Q_{1\text{-}2} + Q_{2\text{-}3}$; $e = W/Q_{in}$	$W = 0.966$ kJ; $Q_{in} = 7.216$ kJ; $e = 0.134 = 13.4\%$

13* ·· An ideal gas ($g = 1.4$) follows the cycle shown in Figure 20-12. The temperature of state 1 is 200 K. Find (*a*) the temperatures of the other three states of the cycle and (*b*) the efficiency of the cycle.

(*a*) Use $PV = nRT$; $T_i = T_1(P_iV_i/P_1V_1)$

$T_1 = 200$ K, $T_2 = 600$ K, $T_3 = 1800$ K, $T_4 = 600$ K

(*b*) Find W = area enclosed by cycle.

$W = 400$ atm·L

Find $Q_{in} = C_v\Delta T_{1\text{-}2} + C_p\Delta T_{2\text{-}3}$

$Q_{in} = (2.5 \times 200 + 3.5 \times 600)$ atm·L = 2600 atm·L

$e = W/Q_{in}$

$e = 400/2600 = 0.154 = 15.4\%$

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14 ··· The *diesel cycle* shown in Figure 20-13 approximates the behavior of a diesel engine. Process *ab* is an adiabatic compression, process *bc* is an expansion at constant pressure, process *cd* is an adiabatic expansion, and process *da* is cooling at constant volume. Find the efficiency of this cycle in terms of the volumes V_a, V_b, V_c, and V_d.

1. Note that $e = 1 - |Q_c|/Q_h$. In the adiabatic portions of the cycle, $Q = 0$. In the portion $b \to c$, $Q = Q_h = C_p(T_c - T_b)$. In the portion $d \to a$, $Q = C_v(T_a - T_d)$ so $|Q_c| = C_v(T_d - T_a)$. Using $g = C_p/C_v$, $e = 1 - (T_d - T_a)/g(T_c - T_b)$. The temperatures are related to the volumes by $T_aV_a^{g-1} = T_bV_b^{g-1}$ and $T_cV_c^{g-1} = T_dV_d^{g-1}$. Also, $V_a = V_d$. Using these relations one can now write the efficiency in terms of volume ratios and the ratio T_b/T_c. One obtains

$$e = 1 - \frac{(V_c/V_a)^{g-1} - (T_b/T_c)(V_b/V_a)^{g-1}}{g[1 - (T_b/T_c)]}.$$

From the ideal gas law, $T_b/T_c = V_b/V_c$. Simplifying the expression, one obtains $e = 1 - \dfrac{(V_c/V_a)^g - (V_b/V_a)^g}{g[(V_c/V_a) - (V_b/V_a)]}$

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15 ·· In the *Stirling cycle* shown in Figure 20-14, process *ab* is an isothermal compression, process *bc* is heating at constant volume, process *cd* is an isothermal expansion, and process *da* is cooling at constant volume. Find the efficiency of the Stirling cycle in terms of the temperatures T_h and T_c and the volumes V_a and V_b.

No work is done in the constant volume segments of the cycle. The work done in the isothermal segments is given by $W_{cd} = nRT_h \ln(V_d/V_c)$ and $W_{ab} = nRT_c \ln(V_b/V_a)$. Since $V_a = V_d$ and $V_b = V_c$, the total work done is $W = nR(T_h - T_c)\ln(V_d/V_c)$.

There is heat input during the segments b→c and c→d. $Q_{bc} = nc_v(T_h - T_c)$; since $\Delta U_{cd} = 0$, $Q_{cd} = nRT_h \ln(V_d/V_c)$. The efficiency is

$$e = \frac{W}{Q_h} = \frac{R(T_h - T_c)\ln(V_d/V_c)}{c_v(T_h - T_c) + RT_h\ln(V_d/V_c)}.$$

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16 ·· The Clausius equation of state is $P(V - bn) = nRT$, where b is a constant. Show that the efficiency of a Carnot cycle is the same for a gas that obeys this equation of state as it is for one that obeys the ideal-gas equation of state, $PV = nRT$.

The Carnot cycle's four segments are: A. An isothermal expansion at $T = T_h$ from V_1 to V_2. B. An adiabatic expansion from V_2 to V_3, at the temperature T_c. C. An isothermal compression from V_3 to V_4. D. An adiabatic compression from V_4 to V_1.

Segment A: $Q_A = W_A = \int_{V_1}^{2} P\,dV = nRT_h \int_{V_1}^{V_2} \frac{dV}{V - bn} = nRT_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right) = Q_h$

Segment C: Following the same procedure as above one obtains $|Q_c| = nRT_c \ln\left(\frac{V_3 - bn}{V_4 - bn}\right)$.

For the complete cycle, $\Delta U = 0$, so $W = Q_h - |Q_c|$.

One obtains for the efficiency $e = \frac{W}{Q_h} = 1 - \frac{T_c}{T_h}\frac{\ln[(V_2 - bn)/(V_1 - bn)]}{\ln[(V_3 - bn)/(V_4 - bn)]}$.

The volumes V_1 and V_4, and V_2 and V_3 are related through the adiabatic process for which $dQ = 0 = dU + dW$.

Thus, $C_v dT + P dV = C_v dT + [nRT/(V - bn)]dV = 0$ and $\int \frac{dT}{T} = -\frac{nR}{C_v}\int \frac{dV}{V - bn} = -(g - 1)\int \frac{dV}{V - bn}$. Therefore,

$T(V - bn)^{g-1}$ = constant, and so $T_h(V_2 - bn)^{g-1} = T_c(V_3 - bn)^{g-1}$ and $T_h(V_1 - bn)^{g-1} = T_c(V_4 - bn)^{g-1}$. It follows that

$\frac{V_2 - bn}{V_1 - bn} = \frac{V_3 - bn}{V_4 - bn}$ and the efficiency is $e = 1 - T_c/T_h$, the same as for the ideal gas.

17* ·· A certain engine running at 30% efficiency draws 200 J of heat from a hot reservoir. Assume that the refrigerator statement of the second law of thermodynamics is false, and show how this engine combined with a perfect refrigerator can violate the heat-engine statement of the second law.

For this engine, $Q_h = 200$ J, $W = 60$ J, and $Q_c = -140$ J. A “perfect” refrigerator would transfer 140 J from the cold reservoir to the hot reservoir with no other effects. Running the heat engine connected to the perfect refrigerator would then have the effect of doing 60 J of work while taking 60 J of heat from the hot reservoir without rejecting any heat, in violation of the heat-engine statement of the second law.

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18 ·· A certain refrigerator takes in 500 J of heat from a cold reservoir and gives off 800 J to a hot reservoir. Assume that the heat-engine statement of the second law of thermodynamics is false, and show how a perfect engine working with this refrigerator can violate the refrigerator statement of the second law.

To remove 500 J from the cold reservoir and reject 800 J to the hot reservoir, 300 J of work must be done on the system. Assuming that the heat-engine statement is false, one could use the 300 J rejected to the hot reservoir to do 300 J of work. Thus, running the refrigerator connected to the “perfect” heat engine would have the effect of transferring 500 J of heat from the cold to the hot reservoir without any work being done, in violation of the refrigerator statement of the second law.

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19 ·· If two adiabatic curves intersect on a *PV* diagram, a cycle could be completed by an isothermal path between the two adiabatic curves shown in Figure 20-15. Show that such a cycle could violate the second law of thermodynamics.

The work done by the system is the area enclosed by the cycle, where we assume that we start with the isothermal expansion. It is only in this expansion that heat is extracted from a reservoir. There is no heat transfer in the adiabatic expansion or compression. Thus we would completely convert heat to mechanical energy, without exhausting any heat to a cold reservoir, in violation of the second law.

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20 · A Carnot engine works between two heat reservoirs at temperatures T_h = 300 K and T_c = 200 K. (*a*) What is its efficiency? (*b*) If it absorbs 100 J from the hot reservoir during each cycle, how much work does it do? (*c*) How much heat does it give off during each cycle? (*d*) What is the COP of this engine when it works as a refrigerator between the same two reservoirs?

(*a*) $e = 1 - T_c/T_h$	$e = 1 - 2/3 = 1/3 = 0.333 = 33.3\%$
(*b*) $W = eQ_h$	$W = 100/3$ J = 33.3 J
(*c*) $\|Q_c\| = Q_h - W$	$\|Q_c\|$ = 66.7 J
(*d*) COP = $\|Q_c\|/W$	COP = 2

21* · A refrigerator works between an inside temperature of 0°C and a room temperature of 20°C. (*a*) What is the largest possible coefficient of performance it can have? (*b*) If the inside of the refrigerator is to be cooled to –10°C, what is the largest possible coefficient of performance it can have, assuming the same room temperature of 20°C?

(*a*) Express the COP in terms of T_h and T_c.	COP = $\|Q_c\|/W = \|Q_c\|/eQ_h = (1 - e)/e = T_c/(T_h - T_c)$
Evaluate COP	COP = 273/20 = 13.7
(*b*) Evaluate COP	COP = 263/30 = 8.77

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22 · An engine removes 250 J from a reservoir at 300 K and exhausts 200 J to a reservoir at 200 K. (*a*) What is its efficiency? (*b*) How much more work could be done if the engine were reversible?

(*a*) Use Equ. 20-2	$e = 1 - 200/250 = 0.2 = 20\%$
(*b*) Use Equ. 20-6 for e_C; find $W = eQ_h$	$e = 1 - 200/300 = 0.333$; W = 83.3 J
Find W for part (*a*) and ΔW	W_a = 50 J; ΔW = 33.3 J

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23 ·· A reversible engine working between two reservoirs at temperatures T_h and T_c has an efficiency of 30%. Working as a heat engine, it gives off 140 J of heat to the cold reservoir. A second engine working between the same two reservoirs also gives off 140 J to the cold reservoir. Show that if the second engine has an efficiency greater than 30%, the two engines working together would violate the heat-engine statement of the second law.

Let the first engine be run as a refrigerator. Then it will remove 140 J from the cold reservoir, deliver 200 J to the hot reservoir, and require 60 J of energy to operate. Now take the second engine and run it between the same reservoirs, and let it eject 140 J into the cold reservoir, thus replacing the heat removed by the refrigerator. If e_2, the efficiency of this engine, is greater than 30%, then Q_{h2}, the heat removed from the hot reservoir by this engine, is $[140/(1 - e_2)]$ J > 200 J, and the work done by this engine is $W = e_2Q_{h2} > 200$ J. The end result of all this is that the second engine can run the refrigerator, replacing the heat taken from the cold reservoir, and do additional mechanical work. The two systems working together then convert heat into mechanical energy without rejecting any heat to a cold reservoir, in violation of the second law.

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24 ·· A reversible engine working between two reservoirs at temperatures T_h and T_c has an efficiency of 20%. Working as a heat engine, it does 100 J of work in each cycle. A second engine working between the same two reservoirs also does 100 J of work in each cycle. Show that if the efficiency of the second engine is greater than 20%, the two engines working together would violate the refrigerator statement of the second law.

If the reversible engine is run as a refrigerator, it will require 100 J of mechanical energy to take 400 J of heat from the cold reservoir and deliver 500 J to the hot reservoir. Now let the second engine, with $e_2 > 0.2$, operate between the same two heat reservoirs and use it to drive the refrigerator. Since $e_2 > 0.2$, this engine will remove less than 500 J from the hot reservoir in the process of doing 100 J of work. The net result is then that no net work is done by the two systems working together, but a finite amount of heat is transferred from the cold to the hot reservoir, in violation of the refrigerator statement of the second law.

25* ·· A Carnot engine works between two heat reservoirs as a refrigerator. It does 50 J of work to remove 100 J from the cold reservoir and gives off 150 J to the hot reservoir during each cycle. Its coefficient of performance COP = Q_c/W = (100 J)/(50 J) = 2. (*a*) What is the efficiency of the Carnot engine when it works as a heat engine between the same two reservoirs? (*b*) Show that no other engine working as a refrigerator between the same two reservoirs can have a COP greater than 2.

(*a*) The efficiency is given by $e = W/Q_h$; e = 50/150 = 0.333 = 33.3%.

(*b*) If COP > 2, then 50 J of work will remove more than 100 J of heat from the cold reservoir and put more than 150 J of heat into the hot reservoir. So running engine (*a*) to operate the refrigerator with a COP > 2 will result in the transfer of heat from the cold to the hot reservoir without doing any net mechanical work in violation of the second law.

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26 ·· A Carnot engine works between two heat reservoirs at temperatures T_h = 300 K and T_c = 200 K. (*a*) What is its efficiency? (*b*) If it absorbs 100 J from the hot reservoir during each cycle, how much work does it do? (*c*) How much heat does it give off in each cycle? (*d*) What is the coefficient of performance of this engine when it works as a refrigerator between these two reservoirs?

(*a*) $e = 1 - T_c/T_h$	$e = 1 - 77/300 = 0.743 = 74.3\%$
(*b*) $W = e Q_h$	$W = 100 \times 0.743$ J = 74.3J
(*c*) $\lvert Q_c \rvert = Q_h - W$	$\lvert Q_c \rvert$ = 25.7 J
(*d*) COP = $\lvert Q_c \rvert / W$	COP = 0.345

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27 ·· In the cycle shown in Figure 20-16, 1 mol of an ideal gas (g = 1.4) is initially at a pressure of 1 atm and a temperature of 0°C. The gas is heated at constant volume to t_2 = 150°C and is then expanded adiabatically until its pressure is again 1 atm. It is then compressed at constant pressure back to its original state. Find (*a*) the temperature t_3 after the adiabatic expansion, (*b*) the heat entering or leaving the system during each process, (*c*) the efficiency of this cycle, and (*d*) the efficiency of a Carnot cycle operating between the temperature extremes of this cycle.

(*a*) 1. Determine P_2.	P_2 = 423/273 atm = 1.55 atm
2. Determine $V_3 = V_1(P_2/P_1)^{1/g}$. Note that V_1 = V/mol at STP = 22.4 L.	$V_3 = 22.4(1.55)^{1/1.4}$ = 30.6 L.
3. Find $T_3 = T_1(V_3/V_1)$	T_3 = 373 K; t_3 = 100°C
(*b*) $Q_{1\text{-}2} = c_v \Delta T$	$Q_{1\text{-}2} = 2.5 \times 8.314 \times 150$ J = 3.12 kJ
$2 \rightarrow 3$ is an adiabatic process	$Q_{2\text{-}3} = 0$
$Q_{3\text{-}1} = c_p \Delta T$	$Q_{3\text{-}1} = -3.5 \times 8.314 \times 100 = -2.91$ kJ
(*c*) $W = \Sigma Q$	W = 0.21 kJ
$e = W/Q_{1\text{-}2}$	e = 0.21/3.12 = 0.0673 = 6.73%
(*d*) $e_{\text{Carnot}} = 1 - T_c/T_h$	$e_{\text{Carnot}} = 1 - T_1/T_2 = 1 - 273/423 = 0.355 = 35.5\%$

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28 ·· A steam engine takes in superheated steam at 270°C and discharges condensed steam from its cylinder at 50°C.

Its efficiency is 30%. (*a*) How does this efficiency compare with the maximum possible efficiency for these temperatures? (*b*) If the useful power output of the engine is 200 kW, how much heat does the engine discharge to its surroundings in 1 h?

(*a*) $e_{max} = 1 - T_c/T_h$ $e_{max} = 1 - 323/534 = 40.5\%$

(*b*) $|Q_c| = (1 - e)Q_h$; $Q_h = W/e = Pt/e$ $Q_h = 7.2 \times 10^8/0.3$ J $= 2.4 \times 10^9$ J; $|Q_c| = 1.68$ GJ

29* · A heat pump delivers 20 kW to heat a house. The outside temperature is –10°C and the inside temperature of the hot-air supply for the heating fan is 40°C. (*a*) What is the coefficient of performance of a Carnot heat pump operating between these temperatures? (*b*) What must be the minimum power of the engine needed to run the heat pump? (*c*) If the COP of the heat pump is 60% of the efficiency of an ideal pump, what must the minimum power of the engine be?

(*a*) COP = $T_c/\Delta T$ (see Problem 21) COP = 263/50 = 5.26

(*b*) Use Equ. 20-10; $P = W/t$ $P = [20/(1 + 5.26)]$ kW = 3.19 kW

(*c*) $P' = P/0.6$ $P' = 3.19/0.6$ kW = 5.32 kW

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30 · Rework Problem 29 for an outside temperature of –20°C.

Follow the procedure of Problem 29 with $T_c = 253$ K, $\Delta T = 60$ K. One obtains (*a*) COP = 4.22; (*b*) $P = 3.83$ kW; (*c*) $P' = 6.39$ kW.

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31 · A refrigerator is rated at 370 W. (*a*) What is the maximum amount of heat it can remove in 1 min if the inside temperature of the refrigerator is 0°C and it exhausts into a room at 20°C? (*b*) If the COP of the refrigerator is 70% of that of an ideal pump, how much heat can it remove in 1 min?

(*a*) Find COP and $Q_c = (\text{COP}) \times W = (\text{COP}) \times Pt$ COP = 273/20 = 13.65;

$Q_c = 13.65 \times 370 \times 60$ J = 303 kJ

(*b*) $Q_c' = 0.7Q_c$ $Q_c' = 212$ kJ

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32 · Rework Problem 31 for a room temperature of 35°C.

Follow the same procedure as in Problem 31, with COP = 273/35 = 7.8. One obtains (*a*) $Q_c = 173$ kJ; (*b*) $Q_c' = 121$ kJ.

33* ·· On a humid day, water vapor condenses on a cold surface. During condensation, the entropy of the water

(*a*) increases.

(*b*) remains constant.

(*c*) decreases.

(*d*) may decrease or remain unchanged.

(*c*)

---

34 · What is the change in entropy of 1 mol of water at 0°C that freezes?

$\Delta S = \Delta Q/T$ $\Delta S = -(18 \times 333.5/273)$ J/K = -22 J/K

---

35 · Two moles of an ideal gas at $T = 400$ K expand quasi-statically and isothermally from an initial volume of 40 L to a final volume of 80 L. (*a*) What is the entropy change of the gas? (*b*) What is the entropy change of the universe for this process?

(*a*) $\Delta S = \Delta Q/T = nR \ln(V_2/V_1)$ $\Delta S = 2 \times 8.314 \times \ln(2)$ J/K = 11.5 J/K

(*b*) The process is reversible. ΔS of universe = 0; (ΔS of outside = −11.5 J/K)

---

36 · The gas in Problem 35 is taken from the same initial state (T = 400 K, V_1 = 40 L) to the same final state (T = 400 K, V_2 = 80 L) by a process that is not quasi-static. (*a*) What is the entropy change of the gas? (*b*) What can be said about the entropy change of the universe?

Since the entropy is a state function, the change in entropy of the gas is as in Problem 35, i.e., 11.5 J/K. In the non-reversible process, the entropy of the universe must increase.

37* · What is the change in entropy of 1.0 kg of water when it changes to steam at 100°C and a pressure of 1 atm?

$\Delta S = \Delta Q/T$ ΔS = 2257/373 kJ/K = 6.05 kJ/K

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38 · Jay approached his guru in a depressed mood. "I want to change the world, but I feel helpless," he said. The guru turned and pushed a 5-kg rock over a ledge. It hit the ground 6 m below and came to rest. "There," said the guru. "I have changed the world." If the rock, the ground, and the atmosphere are all initially at 300 K, calculate the entropy change of the universe.

$\Delta S = \Delta Q/T$; $\Delta Q = mgh$ $\Delta S = 5 \times 9.81 \times 6/300$ J/K = 0.981 J/K

---

39 · What is the change in entropy of 1.0 kg of ice when it changes to water at 0°C and a pressure of 1 atm?

$\Delta S = \Delta Q/T$ ΔS = 333.5/273 kJ/K = 1.22 kJ/K

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40 ·· A system absorbs 200 J of heat reversibly from a reservoir at 300 K and gives off 100 J reversibly to a reservoir at 200 K as it moves from state A to state B. During this process, the system does 50 J of work. (*a*) What is the change in the internal energy of the system? (*b*) What is the change in entropy of the system? (*c*) What is the change in entropy of the universe? (*d*) If the system goes from state A to state B by a nonreversible process, how would your answers for parts (*a*), (*b*), and (*c*) differ?

(*a*) $\Delta U = \Delta Q - W$ ΔQ = 100 J; W = 50 J; ΔU = 50 J

(*b*) $\Delta S = \Delta S_h - \Delta S_c = Q_h/T_h - Q_c/T_c$ ΔS = (200/300 − 100/200) J/K = 0.167 J/K

(*c*) The process is reversible. $\Delta S_u = 0$

(*d*) S_{system} is a state function; the process is irreversible. (*a*) and (*b*) are the same as before. $\Delta S_u > 0$

41* ·· A system absorbs 300 J from a reservoir at 300 K and 200 J from a reservoir at 400 K. It then returns to its original state, doing 100 J of work and rejecting 400 J of heat to a reservoir at a temperature T. (*a*) What is the entropy change of the system for the complete cycle? (*b*) If the cycle is reversible, what is the temperature T?

(*a*) S is a state function of the system. ΔS for complete cycle = 0.

(*b*) $\Delta S = Q_1/T_1 + Q_2/T_2 + Q_3/T_3 = 0$; solve for T_3 1 J/K + 0.5 J/K − (400 J)/T_3 = 0; $T_3 = T$ = 267 K

---

42 ·· Two moles of an ideal gas originally at T = 400 K and V = 40 L undergo a free expansion to twice their initial volume. What is (*a*) the entropy change of the gas, and (*b*) the entropy change of the universe?

(*a*) See Problem 35; ΔS_{gas} = 11.5 J/K; (*b*) It is an irreversible process; $\Delta S_u > 0$. Since no heat is exchanged,

$\Delta S_u = 11.5$ J/K

---

43 ·· A 200-kg block of ice at 0°C is placed in a large lake. The temperature of the lake is just slightly higher than 0°C, and the ice melts. (*a*) What is the entropy change of the ice? (*b*) What is the entropy change of the lake? (*c*) What is the entropy change of the universe (the ice plus the lake)?

(*a*) $\Delta S_{ice} = mL_f/T_f$ — $\Delta S_{ice} = 200 \times 333.5/273$ kJ/K = 244.3 kJ/K

(*b*) $\Delta S_{lake} = -\Delta S_{ice}$ — $\Delta S_{lake} = -244.3$ kJ/K

(*c*) $\Delta S_u = 0$. This is only true under the assumption that both the lake and ice are at exactly the same temperature initially. If that were so, then the ice would not melt. Since the temperature of the lake is slightly greater than that of the ice, the magnitude of the entropy change of the lake is less than 244.3 kJ/K and the entropy change of the universe is greater than zero. The melting of the ice is an irreversible process.

---

44 ·· A 100-g piece of ice at 0°C is placed in an insulated container with 100 g of water at 100°C. (*a*) When thermal equilibrium is established, what is the final temperature of the water? Ignore the heat capacity of the container. (*b*) Find the entropy change of the universe for this process.

(*a*) Use the calorimetry equation. — $100(100 - t) = (100\text{ g})(79.7\text{ cal/g}) + 100(t - 0)$

Solve for t — $t = 10.15^\circ$C

(*b*) $\Delta S_{ice} = mL_f/T_f + mc_p \ln(T_f/T_i)$ — $\Delta S_{ice} = [100 \times 333.5/273 + 100 \times 4.184 \times \ln(283.3/273.15)]$ J/K; $\Delta S_{ice} = 137$ J/K

$\Delta S_{water} = mc_p \ln(T_f/T_i)$ — $\Delta S_{water} = 100 \times 4.184 \ln(283/373) = -116$ J/K

$\Delta S_u = \Delta S_{ice} + \Delta S_{water}$ — $\Delta S_u = 21$ J/K; $\Delta S_u > 0$, process is irreversible.

45* ·· A 1-kg block of copper at 100°C is placed in a calorimeter of negligible heat capacity containing 4 L of water at 0°C. Find the entropy change of (*a*) the copper block, (*b*) the water, and (*c*) the universe.

(a) Use the calorimetry equation to find the final temperature. — $1 \times 0.386(100 - t) = 4 \times 4.184(t - 0)$; $t = 2.26^\circ$C = 275.4 K

Find $\Delta S_{Cu} = m_{Cu}c_{Cu} \ln(T_f/T_i)$ — $\Delta S_{Cu} = 0.386 \ln(275.4/373)$ kJ/K = -117 J/K

(*b*) $\Delta S_w = m_w c_w \ln(T_f/T_i)$ — $\Delta S_w = 4 \times 4.184 \ln(275.4/273.15)$ kJ/K = 137 J/K

(*c*) $\Delta S_u = \Delta S_{Cu} + \Delta S_w$ — $\Delta S_u = 20$ J/K; $\Delta S_u > 0$, the process is irreversible.

---

46 ·· If a 2-kg piece of lead at 100°C is dropped into a lake at 10°C, find the entropy change of the universe.

1. Find the heat lost by the lead — $\Delta Q = 2 \times 0.128 \times 90$ kJ = 23 kJ
2. Find ΔS_w; T_w remains at 283 K — $\Delta S_w = 23/283$ kJ/K = 81.4 J/K
3. Find $\Delta S_{Pb} = m_{Pb}c_{Pb} \ln(T_f/T_i)$ — $\Delta S_{Pb} = 2 \times 0.128 \ln(283/373)$ kJ/K = -70.7 J/K
4. Find $\Delta S_u = \Delta S_w + \Delta S_{Pb}$ — $\Delta S_u = 10.7$ J/K

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47 ·· A 1500-kg car traveling at 100 km/h crashes into a concrete wall. If the temperature of the air is 20°C, calculate the entropy change of the universe.

$\Delta S_u = Q/T = {}_{1/2}mv^2/T$ — $\Delta S_u = {}_{1/2} \times 1500 \times 27.8^2/293$ J/K = 1975 J/K

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48 ·· Find the net change in entropy of the universe when 10 g of steam at 100°C and a pressure of 1 atm are introduced into a calorimeter of negligible heat capacity containing 150 g of water and 150 g of ice at 0°C.

1. Find the heat required to melt 150 g of ice. $Q_1 = 150 \times 333.5$ J = 50 kJ
2. Find the heat released as 10 g of steam at 100°C condense to 0°C. Since $Q_2 < Q_1$, $T_f = 273$ K. $Q_2 = 10 \times 2257 + 10 \times 4.184 \times 100 = 26.75$ kJ
3. Find m', the mass of ice that melts. $m' = 26.75/333.5$ kg = 0.0802 kg = 80.2 g
4. Find $\Delta S_{ice} = Q_2/273$ $\Delta S_{ice} = 98$ J/K
5. Find $\Delta S_{steam} = -m_sL_v/373 + m_sc_w \ln(273/373)$ $\Delta S_{steam} = -22750/373 + 41.84 \ln(0.732) = -74$ J/K
6. $\Delta S_u = \Delta S_{ice} + \Delta S_{steam}$ $\Delta S_u = 24$ J/K

49* ·· If 500 J of heat is conducted from a reservoir at 400 K to one at 300 K, (*a*) what is the change in entropy of the universe, and (*b*) how much of the 500 J of heat conducted could have been converted into work using a cold reservoir at 300 K?

(*a*) $\Delta S_u = \Delta S_h + \Delta S_c = -Q/T_h + Q/T_c$ $\Delta S_u = 500(1/300 - 1/400) = 0.417$ J/K

(*b*) $\varepsilon_{max} = 1 - T_c/T_h$; $W = \varepsilon Q_h$ $\varepsilon_{max} = 0.25$; $W = 0.25 \times 500 = 125$ J

---

50 ·· One mole of an ideal gas first undergoes a free expansion from $V_1 = 12.3$ L and $T_1 = 300$ K to $V_2 = 24.6$ L and $T_2 = 300$ K. It is then compressed isothermally and quasi-statically back to its original state. (*a*) What is the entropy change of the universe for the complete cycle? (*b*) How much work is wasted in this cycle? (*c*) Show that the work wasted is $T\,\Delta S_u$.

(*a*) Although in the adiabatic free expansion no heat is lost by the gas, the process is irreversible and the entropy of the gas increases. In the isothermal reversible process that returns the gas to its original state, the gas releases heat to the surroundings. However, since the process is reversible, the entropy change of the universe is zero. Consequently, the net entropy change is the negative of that of the gas in the isothermal compression.

$\Delta S_{gas} = nR \ln(V_f/V_i) = -\Delta S_u$; $\Delta S_u = nR \ln(V_i/V_f)$ $\Delta S_u = 8.314 \ln(2) = 5.76$ J/K

(*b*) If the initial expansion had been isothermal and reversible, no work would have been done in the cycle.

(*c*) The amount of energy that is dissipated is $T\Delta S_u = 1.73$ kJ.

---

51 · In a reversible adiabatic process,

(*a*) the internal energy of the system remains constant.

(*b*) no work is done by the system.

(*c*) the entropy of the system remains constant.

(*d*) the temperature of the system remains constant.

(*c*)

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52 ·· True or false:

(*a*) Work can never be converted completely into heat.

(*b*) Heat can never be converted completely into work.

(*c*) All heat engines have the same efficiency.

(*d*) It is impossible to transfer a given quantity of heat from a cold reservoir to a hot reservoir.

(*e*) The coefficient of performance of a refrigerator cannot be greater than 1.

(*f*) All Carnot engines are reversible.

(*g*) The entropy of a system can never decrease.

(*h*) The entropy of the universe can never decrease.

(*a*) False. (*b*) True. (*c*) False. (*d*) False. (*e*) False. (*f*) True. (*g*) False. (*h*) True.

53* ·· An ideal gas is taken reversibly from an initial state P_i, V_i, T_i to the final state P_f, V_f, T_f. Two possible paths are (A) an isothermal expansion followed by an adiabatic compression, and (B) an adiabatic compression followed by an isothermal expansion. For these two paths,

(*a*) $\Delta U_A > \Delta U_B$.

(*b*) $\Delta S_A > \Delta S_B$.

(*c*) $\Delta S_A < \Delta S_B$.

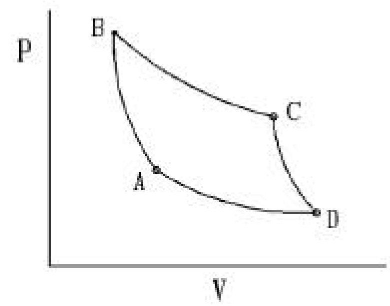
(*d*) none of the above is correct.

(*d*)

---

54 ·· Figure 20-17 shows a thermodynamic cycle on an *ST* diagram. Identify this cycle and sketch it on a *PV* diagram.

The processes A-B and C-D are adiabatic; the processes B-C and D-A are isothermal. The cycle is therefore the Carnot cycle, shown in the adjacent *PV* diagram.



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55 ·· Figure 20-18 shows a thermodynamic cycle on an *SV* diagram. Identify the type of engine represented by this diagram.

Note that A-B is an adiabatic expansion. B-C is a constant volume process in which the entropy decreases; therefore heat is released. C-D is an adiabatic compression. D-A is a constant volume process that returns the gas to its original state. The cycle is that of the Otto engine (see Figure 20-3).

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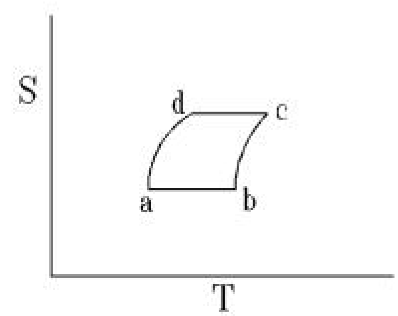
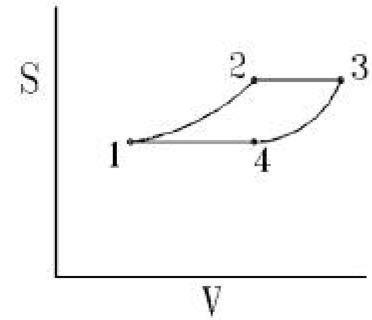
56 ·· Sketch an *ST* diagram of the Otto cycle.

Refer to Figure 20-3. Here a-b is an adiabatic compression, so *S* is constant and *T* increases. Between b and c, heat is added to the system and both *S* and *T* increase. c-d is again isentropic. d-a releases heat and both *S* and *T* decrease. The cycle on an *ST* diagram is sketched in the adjacent figure.



57* ·· Sketch an *SV* diagram of the Carnot cycle.

Referring to Figure 20-8, process 1-2 is an isothermal expansion. In this process heat is added to the system and the entropy and volume increase. Process 2-3 is adiabatic, so S is constant as V increases. Process 3-4 is an isothermal compression in which S decreases and V also decreases. Finally, process 4-1 is adiabatic, i.e., isentropic, and S is constant while V decreases. The cycle is shown in the adjacent SV diagram.



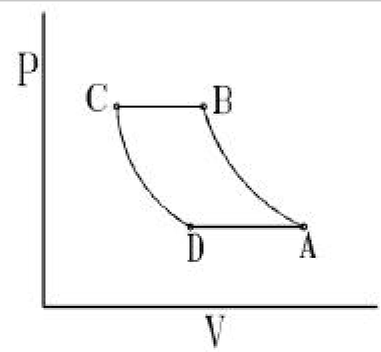
58 ·· Sketch an SV diagram of the Otto cycle.

The SV diagram of the Otto cycle is shown in Figure 20-18. (see Problem 55)

59 ·· Figure 20-19 shows a thermodynamic cycle on an SP diagram. Make a sketch of this cycle on a PV diagram.

Process A-B is at constant entropy, i.e., an adiabatic process in which the pressure increases. Process B-C is one in which P is constant and S decreases; heat is exhausted from the system and the volume decreases.

Process C-D is an adiabatic compression. Process D-A returns the system to its original state at constant pressure. The cycle is shown in the adjacent PV diagram.



60 · An engine with an output of 200 W has an efficiency of 30%. It works at 10 cycles/s. (*a*) How much work is done in each cycle? (*b*) How much heat is absorbed and how much is given off in each cycle?

(*a*) $W/\text{cycle} = P\Delta t$ $\qquad$ $W/\text{cycle} = 200 \times 0.1\text{ J} = 20\text{ J}$

(*b*) $Q_h = W/e$; $|Q_c| = Q_h - W$ $\qquad$ $Q_h = 20/0.3\text{ J} = 66.7\text{ J}$; $|Q_c| = 46.7\text{ J}$

61* · Which has a greater effect on increasing the efficiency of a Carnot engine, a 5-K increase in the temperature of the hot reservoir or a 5-K decrease in the temperature of the cold reservoir?

Let ΔT be the change in temperature and $e = (T_h - T_c)/T_h$ be the initial efficiency. If T_h is increased by ΔT, e', the new efficiency is $e' = (T_h + \Delta T - T_c)/(T_h + \Delta T)$. If T_c is reduced by ΔT, the efficiency is then $e'' = (T_h - T_c + \Delta T)/T_h$. The ratio $e''/e' = T_h/(T_h + \Delta T) > 1$. Therefore, a reduction in the temperature of the cold reservoir by ΔT increases the efficiency more than an equal increase in the temperature of the hot reservoir.

62 · In each cycle, an engine removes 150 J from a reservoir at 100°C and gives off 125 J to a reservoir at 20°C. (*a*) What is the efficiency of this engine? (*b*) What is the ratio of its efficiency to that of a Carnot engine working between the same reservoirs? (This ratio is called the *second law efficiency*.)

(*a*) $e = 1 - |Q_c|/Q_h$ $\qquad$ $e = 1 - 125/150 = 0.167 = 16.7\%$

(*b*) $e_C = 1 - T_c/T_h$ $\qquad$ $e_C = 1 - 293/373 = 0.214 = 21.4\%$; $e/e_C = 0.777$

63 · An engine removes 200 kJ of heat from a hot reservoir at 500 K in each cycle and exhausts heat to a cold reservoir at 200 K. Its efficiency is 85% of a Carnot engine working between the same reservoirs. (*a*) What is the efficiency of this engine? (*b*) How much work is done in each cycle? (*c*) How much heat is exhausted in each cycle?

(*a*) $e = 0.85e_C$; $e_C = 1 - T_c/T_h$ — $e = 0.85(1 - 0.4) = 0.51 = 51\%$

(*b*) $W = eQ_h$ — $W = 0.51 \times 200$ kJ = 102 kJ

(*c*) $|Q_c| = Q_h - W$ — $|Q_c| = 98$ kJ

---

64 · To maintain the temperature inside a house at 20°C, the power consumption of the electric baseboard heaters is 30 kW on a day when the outside temperature is -7°C. At what rate does this house contribute to the increase in the entropy of the universe?

$\Delta S/\Delta t = (\Delta Q/T)/\Delta t$ — $\Delta S/\Delta t = 30/266$ kW/K = 113 W/K

65* ·· The system represented in Figure 20-17 (Problem 54) is 1 mol of an ideal monatomic gas. The temperatures at points A and B are 300 and 750 K, respectively. What is the thermodynamic efficiency of the cyclic process ABCDA?

$e = e_C$ (see Problem 54); $e_C = 1 - T_c/T_h$ — $e = 1 - 300/750 = 0.6 = 60\%$

---

66 ·· A sailor is in a tropical ocean on a boat. She has a 2-kg piece of ice at 0°C, and the temperature of the ocean is T_h = 27°C. Find the maximum work *W* that can be done using the fusion of ice.

$e_{max} = e_C = 1 - T_c/T_h$; $W = e_C Q_h = e_C m_{ice} L_f$ — $W = (1 - 273/300) \times 2 \times 333.5$ kJ = 60 kJ

---

67 ·· (*a*) Which process is more wasteful: (1) a block moving with 500 J of kinetic energy being slowed to rest by friction when the temperature of the atmosphere is 300 K or (2) 1 kJ of heat being conducted from a reservoir at 400 K to one at 300 K? *Hint:* How much of the 1 kJ of heat could be converted into work in an ideal situation? (*b*) What is the change in entropy of the universe for each process?

(*a*) 1. Process (1): All mechanical energy is lost. — Energy loss = 500 J

2. Process (2): Run a Carnot engine. Then $W_{recovered} = (1 - T_c/T_h)Q_h$ — $W_{recovered} = 0.25 \times 1$ kJ = 250 J; (1) is more wasteful of *mechanical* energy. (2) is more wasteful of total energy.

(*b*) $\Delta S_1 = \Delta Q/T$; $\Delta S_2 = \Delta Q(1/T_c - 1/T_h)$ — $\Delta S_1 = 1.67$ J/K; $\Delta S_2 = 0.833$ J/K; $\Delta S_1 > \Delta S_2$.

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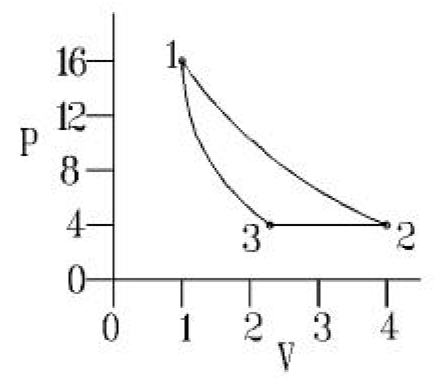
68 ·· Helium gas (g = 1.67) is initially at a pressure of 16 atm, a volume of 1 L, and a temperature of 600 K. It is expanded isothermally until its volume is 4 L and is then compressed at constant pressure until its volume and temperature are such that an adiabatic compression will return the gas to its original state. (*a*) Sketch this cycle on a *PV* diagram. (*b*) Find the volume and temperature after the isobaric compression. (*c*) Find the work done during each cycle. (*d*) Find the efficiency of the cycle.

(*a*) During the isothermal expansion the pressure drops from 16 atm to 4 atm as the volume increases from 1 L to 4 L. The volume at point 3 is determined from the pressure ratio P_1/P_3 and the equation for an adiabatic process, PV^g = constant; $V_3 = V_1(P_1/P_3)^{1/g} = 1(4)^{0.6}$ L = 2.30 L. The complete cycle is shown on the adjacent *PV* diagram; here *P* is in atm and *V* in L.



(*b*) The volume at point 3 is V_3 = 2.30 L (see above).
$T_3 = T_1(V_1/V_3)^{g-1}$ $\qquad$ $T_3 = 600(1/2.3)^{0.667} = 344$ K

(*c*) For process 1-2, $W_{1\text{-}2} = nRT_1 \ln(V_2/V_1) = P_1V_1 \ln(V_2/V_1)$ $\qquad$ $W_{1\text{-}2} = 16 \ln(4)$ atm·L = 22.2 atm·L

For process 2-3, $W_{2\text{-}3} = P_2\Delta V$ $\qquad$ $W_{2\text{-}3} = 4 \times (2.3 - 4)$ atm·L = –6.8 atm·L

For process 3-1, $W_{3\text{-}1} = -C_v\Delta T = 1.5nR\Delta T = -1.5(P_1V_1 - P_3V_3)$ $\qquad$ $W_{3\text{-}1} = -1.5(16 - 9.2)$ atm·L = –10.2 atm·L

(*d*) $Q_{in} = Q_{1\text{-}2}$ = 22.2 atm·L; $W = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$; $e = W/Q_{in}$ $\qquad$ $e = 5.2/22.2 = 0.234 = 23.4\%$

69* ·· A heat engine that does the work of blowing up a balloon at a pressure of 1 atm extracts 4 kJ from a hot reservoir at 120°C. The volume of the balloon increases by 4 L, and heat is exhausted to a cold reservoir at a temperature T_c. If the efficiency of the heat engine is 50% of the efficiency of a Carnot engine working between the same reservoirs, find the temperature T_c.

1. Find *W*. $\qquad$ W = 4 atm·L = 0.404 kJ
2. $e = W/Q_h$; $e_C = 2e = 1 - T_c/T_h$ $\qquad$ $e_C = 2 \times 0.404/4 = 0.202 = 1 - T_c/393$; $T_c = 313.6$ K
3. $t_c = T_c - 273.15$ $\qquad$ $t_c = 40.5^oC$

---

70 ·· Show that the COP of a Carnot refrigerator is related to the efficiency of a Carnot engine by COP = $T_c / (e_C T_h)$.

By definition, COP = $Q_c/W = (Q_h - W)/W = [1 - (W/Q_h)]/(W/Q_h) = (1 - e_C)/e_C = (T_c/T_h)/[1 - (T_c/T_h)] = T_c/e_C T_h$.

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71 ·· A freezer has a temperature T_C = –23°C. The air in the kitchen has a temperature T_h = +27°C. Since the heat insulation is not perfect, some heat flows into the freezer at a rate of 50 W. Find the power of the motor that is needed to maintain the temperature in the freezer.

COP = $T_c/\Delta T$; $P = (dQ_c/dt)/\text{COP} = (dQ_c/dt) \times \Delta T/T_c$ $\qquad$ $P = 50 \times 50/250 = 10$ W

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72 ·· Two moles of a diatomic gas are taken through the cycle ABCA as shown on the *PV* diagram in Figure 20-20. At A the pressure and temperature are 5 atm and 600 K. The volume at B is twice that at A. The segment BC is an adiabatic expansion and the segment CA is an isothermal compression. (*a*) What is the volume of the gas at A? (*b*) What are the volume and temperature of the gas at B? (*c*) What is the temperature of the gas at C? (*d*) What is the volume of the gas at C? (*e*) How much work is done by the gas in each of the three segments of the cycle? (*f*) How much heat is absorbed by the gas in each segment of the cycle? (*g*) What is the thermodynamic efficiency of this cycle?

(*a*) $V_A = nRT_A/P_A$ $\qquad$ $V_A = 2 \times 8.314 \times 600/505$ L = 19.76 L

(*b*) $V_B = 2V_A$; $T_B = 2T_A$	$V_B = 39.52$ L; $T_B = 1200$ K
(*c*) $T_C = T_A$	$T_C = 600$ K
(*d*) $T_B/T_C = (V_C/V_B)^{g-1}$; $V_C/V_B = (T_B/T_C)^{1/(g-1)}$	$V_C = 39.52 \times 2^{2.5} = 224$ L
(*e*) 1. $W_{A\text{-}B} = P_A\Delta V = P_AV_A$	$W_{A\text{-}B} = 5 \times 19.76$ atm·L = 98.8 atm·L = 9.98 kJ
2. $W_{B\text{-}C} = -\Delta U_{B\text{-}C} = -nc_v\Delta T_{B\text{-}C}$	$W_{B\text{-}C} = 2 \times 2.5 \times 8.314 \times 600$ J = 24.9 kJ
3. $W_{C\text{-}A} = nRT \ln(V_A/V_C)$	$W_{C\text{-}A} = 2 \times 8.314 \times 600 \times \ln(19.76/224)$ J = -24.2 kJ
(*f*) 1. $Q_{A\text{-}B} = nc_p\Delta T$	$Q_{A\text{-}B} = 2 \times 3.5 \times 8.314 \times 600 = 34.9$ kJ
2. $Q_{B\text{-}C} = 0$; adiabatic process. $Q_{C\text{-}A} = W_{C\text{-}A}$	$Q_{B\text{-}C} = 0$; $Q_{C\text{-}A} = -24.2$ kJ
(*g*) $e = W/Q_{in}$; $W = W_{A\text{-}B} + W_{B\text{-}C} + W_{C\text{-}A}$	$W = 10.68$ kJ, $Q_{in} = 34.9$ kJ; e _ = 30.6%

73* ·· Two moles of a diatomic gas are carried through the cycle ABCDA shown in the *PV* diagram in Figure 20-21. The segment AB represents an isothermal expansion, the segment BC an adiabatic expansion. The pressure and temperature at A are 5 atm and 600 K. The volume at B is twice that at A. The pressure at D is 1atm. (*a*) What is the pressure at B? (*b*) What is the temperature at C? (*c*) Find the work done by the gas in one cycle and the thermodynamic efficiency of this cycle.

(*a*) 1. $P_B = P_A(V_A/V_B)$	$P_B = 5/2$ atm = 2.5 atm = 252.5 kPa;
2. Find $V_B = nRT_B/P_B$	$V_B = 2 \times 8.314 \times 600/2.525 \times 10^5$ m^3 = 39.5 L
(*b*) 1. Find $V_C = V_B(P_B/P_C)^{1/g}$; $g = 1.4$	$V_C = 39.5 \times 2.5^{0.714}$ L = 76 L
2. $T_C = T_B(P_CV_C/P_BV_B)$	$T_C = 600(76/98.75)$ K = 462 K
(*c*) 1. $W_{A\text{-}B} = nRT_A \ln(V_B/V_A)$	$W_{A\text{-}B} = 2 \times 8.314 \times 600 \times \ln(2)$ J = 6.915 kJ
2. $W_{B\text{-}C} = -nc_v\Delta T$	$W_{B\text{-}C} = 2 \times 2.5 \times 8.314 \times 138$ J = 5.74 kJ
3. $W_{C\text{-}D} = P_C(V_D - V_C)$; $W_{D\text{-}A} = 0$	$W_{C\text{-}D} = 1(19.75 - 76)$ atm·L = -5.68 kJ; $W_{D\text{-}A} = 0$
4. $W = W_{A\text{-}B} + W_{B\text{-}C} + W_{C\text{-}D} + W_{D\text{-}A}$	$W = 6.975$ kJ
5. $Q_{D\text{-}A} = nc_v(T_A - T_D)$; $T_D = T_A/5$	$Q_{D\text{-}A} = 2 \times 2.5 \times 8.314 \times 480$ J = 20 kJ
6. $Q_{in} = Q_{A\text{-}B} + Q_{D\text{-}A}$; $Q_{A\text{-}B} = W_{A\text{-}B}$; $e = W/Q_{in}$	$e = 6.975/26.915 = 0.259 = 25.9\%$

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74 ·· Repeat Problem 72 for a monatomic gas.

(*a*), (*b*), and (*c*) are the same as for Problem 72	$V_A = 19.76$ L; $V_B = 39.52$ L; $T_B = 1200$ K; $T_C = 600$ K
(*d*) $V_C = V_B(T_B/T_C)^{1/(g-1)}$; $g = 5/3$	$V_C = 39.52 \times 2^{1.5} = 111.8$ L
(*e*) 1. $W_{A\text{-}B}$ (see Problem 72)	$W_{A\text{-}B} = 9.98$ kJ
2. $W_{B\text{-}C} = -nc_v\Delta T_{B\text{-}C}$	$W_{B\text{-}C} = 2 \times 1.5 \times 8.314 \times 600 = 14.97$ kJ
3. $W_{C\text{-}A} = nRT_C \ln(V_A/V_C)$	$W_{C\text{-}A} = 2 \times 8.314 \times 600 \times \ln(19.76/111.8)$ J = -17.29 kJ
(*f*) 1. $Q_{A\text{-}B} = nc_p\Delta T$	$Q_{A\text{-}B} = 2 \times 2.5 \times 8.314 \times 600$ J = 24.94 kJ
2. $Q_{B\text{-}C} = 0$; $Q_{C\text{-}A} = W_{C\text{-}A}$	$Q_{B\text{-}C} = 0$; $Q_{C\text{-}A} = -17.29$ kJ
(*g*) $e = W/Q_{in}$; $W = W_{A\text{-}B} + W_{B\text{-}C} + W_{C\text{-}A}$	$W = 7.66$ kJ; $e = 7.66/24.94 = 0.307 = 30.7\%$

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75 ·· Repeat Problem 73 for a monatomic gas.

(*a*) Same as for Problem 73.	$P_B = 2.5$ atm = 252.5 kPa; $V_B = 39.52$ L
(*b*) 1. Find $V_C = V_B(P_B/P_C)^{1/g}$; $g = 5/3$	$V_C = 39.52 \times 2.5^{0.6}$ L = 68.48 L
2. $T_C = T_B(P_CV_C/P_BV_B)$	$T_C = 600(68.48/98.8)$ K = 416 K

(*c*) 1. $W_{A\text{-}B} = nRT_A \ln(V_B/V_A)$ — $W_{A\text{-}B} = 6.915$ kJ (see Problem 73)

2. $W_{B\text{-}C} = -nc_v\Delta T$ — $W_{B\text{-}C} = 2 \times 1.5 \times 8.314 \times 184$ J $= 4.59$ kJ

3. $W_{C\text{-}D} = P_C(V_D - V_C)$; $W_{D\text{-}A} = 0$ — $W_{C\text{-}D} = 1(19.75 - 68.48)$ atm·L $= -4.92$ kJ; $W_{D\text{-}A} = 0$

4. $W = W_{A\text{-}B} + W_{B\text{-}C} + W_{C\text{-}D} + W_{D\text{-}A}$ — $W = 6.585$ kJ

5. $Q_{D\text{-}A} = nc_v(T_A - T_D)$; $T_D = T_A/5$ — $Q_{D\text{-}A} = 2 \times 1.5 \times 8.314 \times 480$ J $= 11.97$ kJ

6. $Q_{in} = Q_{A\text{-}B} + Q_{D\text{-}A}$; $Q_{A\text{-}B} = W_{A\text{-}B}$; $e = W/Q_{in}$ — $e = 6.585/18.89 = 0.349 = 34.9\%$

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76 ·· Compare the efficiency of the Otto engine and the Carnot engine operating between the same maximum and minimum temperatures.

The efficiency of the Otto engine is given in Example 20-2: $e_O = 1 - \dfrac{T_d - T_a}{T_c - T_b}$, where the subscripts refer to the various points of the cycle as shown in Figure 20-3.

Using the relation TV^{g-1} = constant for the adiabatic process, we have $T_c - T_b = T_d(V_d/V_c)^{g-1} - T_a(V_a/V_b)^{g-1}$. In the Otto cycle, $V_a = V_d$ and $V_b = V_c$; thus, $T_c - T_b = (T_d - T_a)(V_a/V_b)^{g-1}$, and $e_O = 1 - (V_b/V_a)^{g-1} = 1 - T_a/T_b$. Note that T_a is the lowest temperature of the cycle, but T_b is not the highest temperature. The high temperature is $T_c = (P_c/P_b)T_b$ which is greater than T_b. A Carnot engine operating between the maximum and minimum temperatures of the Otto will have an efficiency $e_C = 1 - T_a/T_c > 1 - T_a/T_b = e_O$.

77* ·· Compare the efficiency of the Stirling cycle (see Figure 20-14) and the Carnot engine operating between the same maximum and minimum temperatures.

The efficiency of the Sterling cycle, e_S, is given in Problem 15. Using $T_h - T_c = e_C T_h$, that expression can be recast in the form

$$e_S = \frac{e_C}{1 + \dfrac{c_v e_C}{R \ln(V_d/V_c)}}$$

, where V_c and V_d are the volumes indicated in Figure 20-14. Clearly, $e_S < e_C$.

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78 ··· Using the equation for the entropy change of an ideal gas when the volume and temperature change and $TV^{g-1}0$ is a constant, show explicitly that the entropy change is zero for a quasi-static adiabatic expansion from state (V_1,T_1) to state (V_2,T_2).

In general, $\Delta S = C_v \ln(T_2/T_1) + nR \ln(V_2/V_1)$. For the adiabatic process, $(T_2/T_1) = (V_1/V_2)^{g-1}$. So one has $\Delta S = \ln(V_2/V_1)[nR - C_v(g - 1)]$. But $nR = C_p - C_v$ and $gC_v = C_p$. The expression in the square brackets is therefore equal to zero, and $\Delta S = 0$.

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79 ··· (*a*) Show that if the refrigerator statement of the second law of thermodynamics were not true, the entropy of the universe could decrease. (*b*) Show that if the heat-engine statement of the second law were not true, the entropy of the universe could decrease. (*c*) An alternative statement of the second law is that the entropy of the universe cannot decrease. Have you just proved that this statement is equivalent to the refrigerator and heat-engine statements?

(*a*) Suppose the refrigerator statement of the second law is violated in the sense that heat Q_c is taken from the cold reservoir and an equal amount of heat is transferred to the hot reservoir and $W = 0$. The entropy change of the universe is then $\Delta S_u = Q_c/T_h - Q_c/T_c$. Since $T_h > T_c$, $\Delta S_u < 0$, i.e., the entropy of the universe would decrease.

(*b*) In this case, heat Q_h is taken from the hot reservoir and no heat is rejected to the cold reservoir, i.e., $Q_c = 0$, then the entropy change of the universe is $\Delta S_u = -Q_h/T_h + 0$ which is negative. Again, the entropy of the universe would decrease.

(*c*) The heat-engine and refrigerator statements of the second law only state that *some* heat must be rejected to a cold reservoir and *some* work must be done to transfer heat from the cold to the hot reservoir, but these statements do not specify the minimum amount of heat rejected or work that must be done. The statement $\Delta S_u \geq 0$ is more restrictive. The heat-engine and refrigerator statements in conjunction with the Carnot efficiency are equivalent to $\Delta S_u \geq 0$.

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80 ··· Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in Figure 20-22. The efficiencies of the engines are e_1 and e_2 respectively. Show that the net efficiency of the combination is given by $e_{net} = e_1 + (1 - e_1)e_2$.

Referring to Figure 20-22, $e_1 = W_1/Q_h$ and $e_2 = W_2/Q_m$. The overall efficiency is $e_{net} = (W_1 + W_2)/Q_h$ $= e_1 + (Q_m/Q_h)\, e_2$. Since Q_m is the heat rejected by engine 1, $Q_m/Q_h = 1 - e_1$. So $e_{net} = e_1 + (1 - e_1)\, e_2$.

81* ··· Suppose that each engine in Figure 20-22 is an ideal reversible heat engine. Engine 1 operates between temperatures T_h and T_m and Engine 2 operates between T_m and T_c, where $T_h > T_m > T_c$. Show that

$$e_{net} = 1 - \frac{T_c}{T_h}$$

This means that two reversible heat engines in series are equivalent to one reversible heat engine operating between the hottest and coldest reservoirs.

Since the engines are ideal reversible engines, $e_1 = 1 - T_m/T_h$ and $e_2 = 1 - T_c/T_m$. Using the expression derived in Problem 80 one obtains $e_{net} = 1 - T_c/T_h$.

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82 ··· The cooling compartment of a refrigerator and its contents are at 5°C and have an average heat capacity of 84 kJ/K. The refrigerator exhausts heat to the room, which is at 25°C. What minimum power will be required by the motor that runs the refrigerator if the temperature of the cooling compartment and its contents is to be reduced by 1 C° in 1 min?

1. Determine the COP = $T_c/\Delta T$	COP = 278/20 = 13.9
2. $P = W/t = (\lvert Q_h \rvert /t)/(1 + \text{COP})$; $\lvert Q_h \rvert = C\Delta T = C$	$P = 84 \times 10^3/(60 \times 14.9)$ W = 94 W

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83 ··· An insulated container is separated into two chambers of equal volume by a thin partition. On one side of the container there are twelve ^{131}Xe atoms, on the other side there are twelve ^{132}Xe atoms. The partition is then removed. Calculate the change in entropy of the system after equilibrium has been established (that is, when the ^{131}Xe and ^{132}Xe atoms are evenly distributed throughout the total volume).

Removing the partition is equivalent to free expansion; i.e., ^{131}Xe and ^{132}Xe each expand freely into the other volume previously unoccupied by these gases. The change in entropy for each gas is given by Equ. 20-18, where here $V_2/V_1 = 2$. We also recall that $R = N_A k$ and $n = N/N_A$, where N_A is Avogadro's number. Thus, $nR = Nk$. Here $N = 12$. We obtain $\Delta S = 2 \times 12 \times 1.38 \times 10^{-23} \ln(2)$ J/K $= 2.3 \times 10^{-22}$ J/K.
