Elementary Statistics

A Step by Step Approach
Sixth Edition

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CHAPTER 4

Probability and Counting Rules

Objectives

4-1

- Determine sample spaces and find the probability of an event using classical probability or empirical probability.
- Find the probability of compound events using the addition and the multiplication rules.
- Find the conditional probability of an event.
- Determine the number of outcomes of a sequence of events using a tree diagram.

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Objectives

4-2

- Find the total number of outcomes in a sequence of events using the fundamental counting rule.
- Find the number of ways *r* objects can be selected from *n* objects using the permutation and the combination rules.
- Find the probability of an event using the counting rules.

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Introduction

4-3

- *Probability* as a general concept can be defined as the chance of an event occurring. In addition to being used in games of chance, probability is used in the fields of insurance, investments, and weather forecasting, and in various areas.
- Rules such as the fundamental counting rule, combination rule and permutation rule allow us to count the number of ways in which events can occur.

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Basic Concepts

4-4

- A *probability experiment* is a chance process that leads to well-defined results called outcomes.
- An *outcome* is the result of a single trial of a probability experiment.
- A *sample space* is the set of all possible outcomes of a probability experiment.
- An *event* consists of a set of outcomes of a probability experiment. An event with one outcome is called a simple event and with more than one outcome is called compound event.

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4-5

Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl.

There are two gender and three children, so there are $2^3 = 8$ possibilities as shown here,

BGG GBG GGB GGG GBB BGB BBG BBB

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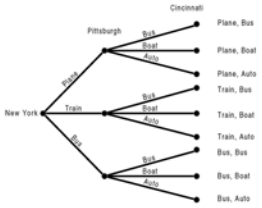
Basic Concepts

4-6

- A *tree diagram* is a device used to list all possibilities of a sequence of events in a systematic way.



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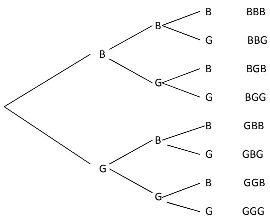
4-7

Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl. Use a tree diagram to find the sample space for the gender of the three children.



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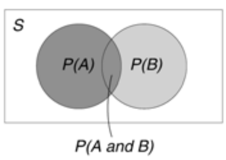
Basic Concepts

4-8

- *Equally likely events* are events that have the same probability of occurring.
- *Venn diagrams* are used to represent probabilities pictorially.



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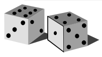
Classical Probability

4-9

- *Classical probability* uses sample spaces to determine the numerical probability that an event will happen.
- *Classical probability* assumes that all outcomes in the sample space are equally likely to occur.

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{Number of outcomes in } E}{\text{total number of outcomes in the sample space}}$$



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4-10

If a family has three children, find the probability that all the children are girls.

There are two gender and three children, so there are $2^3 = 8$ possibilities as shown here,

BGG GBG GGB GGG GBB BGB BBG BBB

Hence, there is one way in eight possibilities for all three children to be girls,

$$P(GGG) = \frac{1}{8}$$

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Rounding Rule for Probabilities

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Note: Probabilities should be expressed as reduced fractions or rounded to two or three decimal places. When the probability of an event is an extremely small decimal, it is permissible to round the decimal to the first nonzero digit after the decimal point.

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Probability Rules

4-12

1. The probability of an event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \le P(E) \le 1$.
2. If an event E cannot occur (i.e., the event contains no members in the sample space), the probability is zero.
3. If an event E is certain, then the probability of E is 1.
4. The sum of the probabilities of the outcomes in the sample space is 1.

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4-13

When a single die is rolled, find the probability of getting a 9.

Since the sample space is 1,2,3,4,5 and 6, it is impossible to get a 9. Hence, $P(9) = \frac{0}{6} = 0$

When a single die is rolled, what is the probability of getting a number less than 7?

Sine all outcomes (1,2,3,4,5 and 6) are less than 7, $P(\text{number less than 7}) = \frac{6}{6} = 1$

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Complementary Events

4-14

- The *complement of an event* E is the set of outcomes in the sample space that are not included in the outcomes of event E. The complement of E is denoted by $\overline{E}$.
- Rule for Complementary Events

$$P(\overline{E}) = 1 - P(E), \quad P(E) = 1 - P(\overline{E})$$

or

$$P(E) + P(\overline{E}) = 1$$

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4-15

Find the complement of each event.

a. Rolling a die and getting a 4.

Getting a 1,2,3,5 or 6

b. Selecting a letter of the alphabet and getting a vowel.

Getting a consonant

c. Selecting a month and getting a month that begins with a J.

Getting February, March, April, May, August, September, October, November or December

d. Selecting a day of the week and getting a weekday.

Getting Saturday or Sunday

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4-16

If the probability that a person lives in an industrialized country of the world is $\frac{1}{5}$, find the probability that a person does not live in an industrialized country.

$$P(\text{not living in an industrialized country}) = 1 - P(\text{living in an industrialized country})$$
$$= 1 - \frac{1}{5} = \frac{4}{5}$$

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Empirical Probability

4-17

- *Empirical probability* relies on actual experience to determine the likelihood of outcomes.
- Given a frequency distribution, the probability of an event being in a given class is:

$$P(E) = \frac{\text{frequency for the class}}{\text{total frequencies in the distribution}} = \frac{f}{n}$$

- *Subjective probability* uses a probability value based on an educated guess or estimate, employing opinions and inexact information.

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4-18

In a sample of 50 people, 21 had type O blood. 22 had type A, 5 had type B blood and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

$$P(O) = \frac{f}{n} = \frac{21}{50}$$

b. A person has type A or type B blood.

$$P(A \text{ or } B) = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$$

c. A person has neither type A nor type O blood.

$$P(\text{neither } A \text{ nor } O) = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$$

d. A person does not have type AB blood.

$$P(\text{not } AB) = 1 - P(AB) = 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$$

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Mutually Exclusive Events

4-19

- Two events are *mutually exclusive* if they cannot occur at the same time (i.e., they have no outcomes in common).
- The probability of two or more events can be determined by the *addition rules*.

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4-20

Determine which events are mutually exclusive and which are not, when a single die is rolled.

a. Getting an odd number and getting an even number.

The events are mutually exclusive, since the first event can be 1, 3 or 5 and the second event can be 2, 4 or 6.

b. Getting a 3 and getting an odd number.

The events are not mutually exclusive, since the first event is a 3 and then second event can be 1, 3 or 5. Hence, 3 is contained in both events.

c. Getting an odd number and getting a number less than 4.

The events are not mutually exclusive, since the first event can be 1, 3 or 5 and the second event can be 1, 2 or 3. Hence, 1 and 3 are contained in both events.

d. Getting a number greater than 4 and getting a number less than 4.

The events are mutually exclusive, since the first event can be 5 or 6 and the second event can be 1, 2 or 3.

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Addition Rules

4-21

- Addition Rule 1—When two events A and B are mutually exclusive, the probability that A or B will occur is:

$$P(A \text{ or } B) = P(A) + P(B)$$

- Addition Rule 2—If A and B are *not* mutually exclusive, then:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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4-22

A box contains 3 glazed doughnuts, 4 jelly doughnuts and 5 chocolate doughnuts. If a person selects a doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut.

The total number of doughnuts in the box is 12 and the event are mutually exclusive, so

$$P(\text{glazed or chocolate}) = P(\text{glazed}) + P(\text{chocolate}) = \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$$

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4-23

A day of the week is selected at random. Find the probability that it is a weekend day (Thursday or Friday)

The total number of days in week is 7 and the event are mutually exclusive, so

$$P(\text{Thursday or Friday}) = P(\text{Thursday}) + P(\text{Friday}) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$$

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4-24

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff is selected, find the probability that the subject is a nurse or a male.

The events are not mutually exclusive and the sample space is

Staff	Female	Males	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

$$P(nurse\ or\ male) = P(nurse) + P(male) - P(male\ nurse)$$
$$= \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \frac{10}{13}$$

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Independent and Dependent Events

4-25

- Two events A and B are ***independent*** if the fact that A occurs does not affect the probability of B occurring.
- When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be ***dependent***.

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Multiplication Rules

4-26

- The ***multiplication rules*** can be used to find the probability of two or more events that occur in sequence.
- Multiplication Rule 1—When two events are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

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4-27

An urn contains 3 red balls, 2 blue balls and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a. Selecting 2 blue balls

$$P(blue\ and\ blue) = P(blue) \times P(blue) = \frac{2}{10} \times \frac{2}{10} = \frac{4}{100} = \frac{1}{25}$$

b. Selecting 1 blue ball and then 1 white ball

$$P(blue\ and\ white) = P(blue) \times P(white) = \frac{2}{10} \times \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$$

c. Selecting 1 red ball and then 1 blue ball

$$P(red\ and\ blue) = P(red) \times P(blue) = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$$

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4-28

Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.

Let C denote red-green color blindness. Then

$$P(C\ and\ C\ and\ C) = P(C) \times P(C) \times P(C)$$
$$= 0.09 \times 0.09 \times 0.09 = 0.000729$$

Hence, the rounded probability is 0.0007

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Multiplication Rules

4-29

- The *conditional probability* of an event B in relationship to an event A is the probability that event B occurs after event A has already occurred. The notation for conditional probability is P(B|A)
- Multiplication Rule 2—When two events are dependent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

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4-30

A person owns a collection of 30 CDs, of which 5 are Holy Quran. If 2 CDs are selected at random, find the probability that both are Holy Quran.

Since the events are dependent,

$$P(CD_1 \text{ and } CD_2) = P(CD_1) \times P(CD_2 \mid CD_1)$$
$$= \frac{5}{30} \times \frac{4}{29} = \frac{20}{870} = \frac{2}{87}$$

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4-31

The World Wide Insurance Company found that 53% of the residents of a city had homeowner's insurance (H) with the company. Of these clients, 27% also had automobile insurance (A) with the company. If a resident is selected at random, find the probability that the resident has both homeowner's and automobile insurance with the World Wide Insurance Company.

Thus, $P(H \text{ and } A) = P(H) \times P(A \mid H) = 0.53 \times 0.27 = 0.1431$

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4-32

Box 1 contains 2 red balls and 1 blue ball. Box 2 contains 3 blue balls and 1 red ball. A coin is tossed. If it falls heads up, box 1 is selected and a ball is drawn. If it falls tails up, box 2 is selected and a ball is drawn. Find the probability of selecting a red ball.

Thus,

$$P(red) = P(Box_1) \times P(red \mid Box_1) + P(Box_2) \times P(red \mid Box_2)$$
$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{4} = \frac{2}{6} + \frac{1}{8} = \frac{8}{24} + \frac{3}{24} = \frac{11}{24}$$

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Formula for Conditional Probability

4-33

- The probability that the second event B occurs given that the first event A has occurred can be found by dividing the probability that both events occurred by the probability that the first event has occurred.
- The formula is:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

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4-34

A box contains black chips and white chips. A person selects two chips without replacement. If the probability of selecting a black chip and a white chip is $15/56$, and the probability of selecting a black chip on the first draw is $3/8$, find the probability of selecting the white chip on the second draw, given that the first chip selected was a black chip.

Let B= selecting a black chip and W= selecting a white chip, then

$$P(W|B) = \frac{P(B \text{ and } W)}{P(B)} = \frac{15/56}{3/8} = \frac{15}{56} \times \frac{8}{3} = \frac{5}{7}$$

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A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes (Y)	No (N)	Total
Male (M)	32	18	50
Female (F)	8	42	50
Total	40	60	100

Find these probabilities

a. The respondent answered yes, given that the respondent was a female

$$P(Y|F) = \frac{P(Y \text{ and } F)}{P(F)} = \frac{8/100}{50/100} = \frac{8}{100} \times \frac{100}{50} = \frac{4}{25}$$

b. The respondent was a male, given that the respondent answered no.

$$P(M|N) = \frac{P(M \text{ and } N)}{P(N)} = \frac{18/100}{60/100} = \frac{18}{100} \times \frac{100}{60} = \frac{3}{10}$$

c. The respondent is female or answered no.

$$P(F \text{ or } N) = P(F) + P(N) - P(F \text{ and } N) = \frac{50}{100} + \frac{60}{100} - \frac{42}{100} = \frac{68}{100}$$

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Counting Rule

4-36

- The multiplication rule can be used to determine the total number of outcomes in a sequence of events.
- Fundamental counting rule: In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3 and so forth, the total number of possibilities of the sequence will be:

$$k_1 \times k_2 \times k_3 \times \cdots \times k_n$$

- Note: "And" in this case means to multiply.

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4-37

A paint manufacturer whishes to manufacture several different paints. The categories include

Color	Red, blue, white, black, green, brown, yellow
Type	Latex, oil
Texture	Flat, semi gloss, high gloss
Use	Outdoor, indoor

How many different kinds of paint can be made if a person can select one color, one type, one texture and one use?

Since there are 7 color choices, 2 type choices, 3 texture choices and 2 use choices, then the total number of possible different paints is $7 \times 2 \times 3 \times 2 = 84$

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4-38

The digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are to be used in a four-digit ID card. How many different cards are possible if repetitions are permitted?

Since there are 4 spaces to fill and 10 choices for each space, then the number of possible different cards is

$$10 \times 10 \times 10 \times 10 = 10^4 = 10000$$

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Permutations

4-39

- A *permutation* is an arrangement of n objects in a specific order.
- The arrangement of n objects in a specific order using r objects at a time is called a *permutation of n objects taking r objects at a time*. It is written as ${}_nP_r$, and the formula is:

$${}_nP_r = \frac{n!}{(n-r)!}$$

where

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$$
$$0! = 1$$

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4-40

Suppose a business owner has a choice of five locations in which to establish his business. He decide to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can he rank the five locations?

Since there is 5 choices for the first location, 4 choices for the second location, 3 choices for the third location, 2 choices for the fourth location and 1 choice for the last location, then the number of ways is

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

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4-41

A television news director wishes to use three news stories on an evening show. One story will be the lead story, one will be the second story and the last will be a closing story. If the director has a total of eight stories to choose from, how many possible ways can the program be set up?

Since the order is important, then the number of ways to set up the program is

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5!}{5!} = 8 \times 7 \times 6 = 336$$

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Combinations

4-42

- A selection of distinct objects without regard to order is called a *combination*.
- The number of combinations of r objects selected from n objects is denoted ${}_nC_r$ and is given by the formula:

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

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4-43

How many combination of 4 objects are there, taken 2 at a time?

Since this is a combination problem, then

$$ {}_4C_2 = \frac{4!}{(4-2)!\times 2!} = \frac{4!}{2!\times 2!} = \frac{4\times 3\times 2!}{2!\times 2\times 1} = \frac{4\times 3}{2} = 6 $$

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4-44

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Here, one must select 3 women from 7 women and select 2 men from 5 men. Then using the fundamental counting rule we can find number of different possibilities

$$ {}_7C_3 \times {}_5C_2 = \frac{7!}{(7-3)!\times 3!} \times \frac{5!}{(5-2)!\times 2!} = \frac{7\times 6\times 5\times 4!}{4!\times 3!} \times \frac{5\times 4\times 3!}{3!\times 2!} $$

$$ = \frac{7\times 6\times 5\times 5\times 4}{3\times 2\times 2} = 350 $$

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4-45

A box contains 24 transistors, 4 of which are defective. If 4 are sold at random, find the following probabilities.

a. Exactly 2 are defective.

$$P(\text{exactly 2 defectives}) = \frac{(\text{\# of selecting 2 defectives}) \times (\text{\# of selecting 2 nondefectives})}{\text{\# of selecting 4 transistors}}$$

$$= \frac{{}_4C_2 \times {}_{20}C_2}{{}_{24}C_4} = \frac{\left(\frac{4!}{2!\times 2!}\right)\times\left(\frac{20!}{18!\times 2!}\right)}{\frac{24!}{20!\times 4!}} = \frac{1140}{10626} = \frac{190}{1771}$$

b. None is defective.

$$P(\text{no defective}) = \frac{\text{\# of selecting no defective}}{\text{\# of selecting 4 transistors}} = \frac{{}_{20}C_4}{{}_{24}C_4} = \frac{1615}{3542}$$

c. All are defective.

$$P(\text{all defective}) = \frac{\text{\# of selecting defective}}{\text{\# of selecting 4 transistors}} = \frac{{}_4C_4}{{}_{24}C_4} = \frac{1}{10626}$$

d. At least 1 is defective.

$$P(\text{at least 1 defective}) = 1 - P(\text{no defective}) = 1 - \frac{{}_{20}C_4}{{}_{24}C_4} = 1 - \frac{1615}{3542} = \frac{1927}{3542}$$

Notes

4-46

A store has 6 TV Graphic magazines and 8 Newstime magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

Thus, $P(1\,TV\ Graphic\ and\ 1\,Newstime) = \frac{{}_6C_1 \times {}_8C_1}{{}_{14}C_2} = \frac{6\times 8}{91} = \frac{48}{91}$

A box contains 2 red balls and 8 blue balls. If 3 balls are selected, what is the probability of having at least one red ball among the selected balls?

Thus, we need to find the probability of having [(1 red ball and 2 blue balls) or (2 red balls and 1 blue ball)].

$\text{\# of selecting 1 red and 2 blue} = {}_2C_1 \times {}_8C_2 = 2 \times 28 = 56$

$\text{\# of selecting 2 red and 1 blue} = {}_2C_2 \times {}_8C_1 = 1 \times 8 = 8$

$\text{\# of having at least 1 red among the 3 chosen balls} = 56 + 8 = 64$

$\text{\# of selecting 3 balls} = {}_{10}C_3 = 120$

So, the probability is $\frac{64}{120} = \frac{8}{15}$

Notes

4-47

A combination lock consists of the 26 letters of the alphabet. If a 3-letter combination is needed, find the probability that the combination will consist of the letters ABC in that order. The same letter can be used more than once.

Since repetitions are permitted, there are $26 \times 26 \times 26 = 17576$ different possible combinations. And since there is only one ABC combination, the probability is

$$P(ABC) = \frac{1}{26^3} = \frac{1}{17576}$$

Notes

4-48

There are 8 married couples in a tennis club. If 1 man and 1 woman are selected at random to plan the summer tournament, find the probability that they are married to each other.

Since there are 8 ways to select a man and 8 ways to select a woman, then there are 64 ways to select 1 man and 1 woman. Since there are 8 married couples, the probability is

$$P(\text{selecting married couples}) = \frac{8}{64} = \frac{1}{8}$$

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Summary

4-49

- The three types of probability are *classical*, *empirical*, and *subjective*.
- Classical probability uses sample spaces.
- Empirical probability uses frequency distributions and is based on observations.
- In subjective probability, the researcher makes an educated guess about the chance of an event occurring.

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Summary

4-50

- An event consists of one or more outcomes of a probability experiment.
- Two events are said to be mutually exclusive if they cannot occur at the same time.
- Events can also be classified as independent or dependent.
- If events are independent, whether or not the first event occurs does not affect the probability of the next event occurring.

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Summary

4-51

- If the probability of the second event occurring is changed by the occurrence of the first event, then the events are dependent.
- The *complement* of an event is the set of outcomes in the sample space that are not included in the outcomes of the event itself.
- Complementary events are mutually exclusive.

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Summary

4-52

Rule	Definition
Multiplication rule $k_1 \cdot k_2 \cdot k_3 \cdots k_n$	The number of ways a sequence of n events can occur; if the first event can occur in k_1 ways, the second event can occur in k_2 ways, etc.
Permutation rule ${}_nP_r = \frac{n!}{(n-r)!}$	The arrangement of n objects in a specific order using r objects at a time
Combination rule ${}_nC_r = \frac{n!}{(n-r)!r!}$	The number of combinations of r objects selected from n objects (order is not important)

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