بسم الله الرحمن الرحيم

King Abdulaziz University

Exam for Chapter 0

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على ثلاثة وثلاثون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus I
Math110

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Enter Name:

I.D. Number:

Start Answer each of the following.

1. Solve the inequality $2x - 3 < -7$

- $(-\infty, -2)$
- $(-\infty, -2]$
- $(-2, \infty)$
- $[-2, \infty)$

2. Solve the inequality $-2 < 2 - 2x < 3$

- $[-2, 1/2]$
- $[-1/2, 2]$
- $(-2, 1/2)$
- $(-1/2, 2)$

3. Solve the inequality $x^2 + 4x + 3 < 0$

- ☐ $(-\infty, -3) \cup (-1, \infty)$
- ☐ $(-\infty, -3] \cup [-1, \infty)$
- ☐ $(-3, -1)$
- ☐ $[-3, -1]$

4. Solve $|2x + 1| < 1$

- ☐ $[-1, 0]$
- ☐ $(-1, 0)$
- ☐ $(-\infty, -1) \cup (0, \infty)$
- ☐ $(-\infty, -1] \cup [0, \infty)$

5. Solve $|2x+1| > 2$

- ☐ $(-\infty, -3/2) \cup (1/2, \infty)$
- ☐ $(-\infty, -3/2] \cup [1/2, \infty)$
- ☐ $(-3/2, 1/2)$
- ☐ $[-3/2, 1/2]$

6. Solve $\dfrac{x+2}{x-2} \geq 0.$

- ☐ $(-\infty, -2) \cup (2, \infty)$
- ☐ $(-\infty, -2] \cup (2, \infty)$
- ☐ $(-2, 2)$
- ☐ $(-\infty, -2] \cup [2, \infty)$

7. The distance between the points $(5,2),(1,-1)$ is

- [ ] 5
- [ ] -5
- [ ] $\sqrt{5}$
- [ ] $-\sqrt{5}$

8. Is the points $(1,1),(3,4),(06)$ forms the vertices of a right triangle.

- [ ] No
- [ ] Yes

9. Determine if the given points are collinear. $(2,1), (0,2), (4,0)$

☐ No

☐ Yes

10. Find the slope m of the line through the points $(3,-6)$ and $(1,-1)$.

☐ $m = \frac{5}{2}$

☐ $m = \frac{-2}{5}$

☐ $m = \frac{-5}{2}$

☐ $m = \frac{2}{5}$

11. Find a second point on the line with slope $\frac{-1}{2}$ and passes through $(1, 2)$.

- [ ] $(3, 1)$
- [ ] $(0, 4)$
- [ ] $(3, 3)$
- [ ] $(4, 0)$

12. Determine if the two lines are parallel, perpendicular, or neither. $y – 3x – 1 = 0$ and $9y + 3x = -6$.

- [ ] perpendicular
- [ ] parallel
- [ ] neither

13. Find an equation of the line through the point $(3,1)$ and perpendicular to the line $y-2x=1$.

- ☐ $y = -2x + 7$
- ☐ $y = 2x - 5$
- ☐ $y = \frac{1}{2}x - \frac{1}{2}$
- ☐ $y = \frac{-1}{2}x + \frac{5}{2}$

14. Find the domain of the function $f(x) = x - 1$.

- ☐ $\mathbb{R}$
- ☐ $(-\infty,1)\cup(1,\infty)$
- ☐ $\mathbb{R}\setminus\{-1\}$
- ☐ $(-\infty,1]$

15. Find the domain of the function $f(x) = \sqrt{2x-3}$.

☐ $(3/2, \infty)$

☐ $\mathbb{R} \setminus \{3/2\}$

☐ $(-\infty, 3/2) \cup (3/2, \infty)$

☐ $[3/2, \infty)$

16. Find the domain of the function

$$f(x) = \frac{5x+1}{x^2+4}.$$

☐ $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

☐ $\mathbb{R}$

☐ $(-\infty, 2) \cup (2, \infty)$

☐ $(-\infty, 2) \cup (2, \infty)$

17. Find the domain of the function $f(x) = \sqrt[3]{x^2 - 1}$.

- ☐ $(-\infty, -1) \cup (1, \infty)$
- ☐ $\mathbb{R} \setminus \{-1, 1\}$
- ☐ $[-1, 1]$
- ☐ $\mathbb{R}$

18. Find the domain of the function $f(x) = \dfrac{5x + 1}{x^2 - x - 6}$.

- ☐ $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
- ☐ $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
- ☐ $(-\infty, -2) \cup (3, \infty)$
- ☐ $(-\infty, -3] \cup [2, \infty)$

19. Find all intercepts of the graph $f(x) = \dfrac{2x-1}{x^2-4}$.

☐ x-inter(s): $x = 1/2$; y-inter:$y = 1/4$.

☐ x-inter(s): $x = \pm 2, -1/2$; y-inter:$y = -1/4$.

☐ x-inter(s): $x = 1/2$; y-inter:$y = 1/4$.

☐ x-inter(s): $x = -2$; y-inter:$y = -1/4$.

20. Find all zeros of $f(x) = x^2 - 2x - 8$.

☐ $-2, -4$

☐ $2, -4$

☐ $2, 4$

☐ $-2, 4$

21. Find the points of intersection of $y = x^2 - 2x + 4$ and $y = 3x - 2$.

- ☐ $(-2, -8), (-3, -11)$
- ☐ $(-3, -11)$
- ☐ $(-2, -8)$
- ☐ $(2, 4), (3, 7)$

22. Covert the $120°$ to radian

- ☐ $2\pi/3$
- ☐ $\pi/3$
- ☐ $4\pi/3$
- ☐ $\pi/6$

23. Covert the $\frac{4\pi}{3}$ to degree

- ☐ 140°
- ☐ 160°
- ☐ 240°
- ☐ 210°

24. $\cos\left(\dfrac{\pi}{4}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{\sqrt{2}}$
- ☐ $-\frac{1}{\sqrt{2}}$
- ☐ $\frac{\sqrt{3}}{2}$

25. $\sin\left(\frac{5\pi}{3}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{\sqrt{3}}{2}$
- ☐ $-\frac{\sqrt{3}}{2}$
- ☐ $-\frac{1}{2}$

26. $\cos^2 x =$

- ☐ $\frac{1-\cos(2x)}{2}$
- ☐ $\frac{1+\cos(2x)}{2}$
- ☐ $\frac{1-\sin(2x)}{2}$
- ☐ $\frac{1+\sin(2x)}{2}$

27. $\cos(x+\pi) =$

- ☐ $-\sin x$
- ☐ $\sin x$
- ☐ $-\cos x$
- ☐ $\cos x$

28. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$, then $(g - f)(x) =$.

- ☐ $\sqrt{x-1} - x^2 - 1$
- ☐ $x^2 + 1 - \sqrt{x-1}$
- ☐ $\sqrt{x-1} - x^2 + 11$
- ☐ $\sqrt{x-1} + x^2 + 1$

29. If $f(x) = \sqrt{x-3}$ and $g(x) = x - 5$, then domain $\left(\frac{f}{g}\right)$ is

- ☐ $\mathbb{R}$
- ☐ $(-\infty, 3)$
- ☐ $[3, 5) \cup (5, \infty)$
- ☐ $[3, \infty)$

30. Find the domain of the function

$$f(x) = \sqrt{25 - x^2} - \sqrt{x^2 - 4}.$$

- ☐ $\mathbb{R}$
- ☐ $[-5, -2] \cup [2, 5]$
- ☐ $\mathbb{R} \setminus \{\pm 2, \pm 5\}$
- ☐ $(-\infty, -5] \cup [5, \infty)$

31. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $(f \circ g)(x) =$

- ☐ $x + 2$
- ☐ $\sqrt{x^2 + 2}$
- ☐ $\sqrt{x+3} - 1$
- ☐ $\sqrt{\sqrt{x^2 - 1} + 2}$

32. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $D(f \circ g) =$

- ☐ $(-\infty, -3) \cup (-3, \infty)$
- ☐ $\mathbb{R}$
- ☐ $(-3, \infty)$
- ☐ $[-3, \infty)$

33. Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x-1}$, then $(g \circ f)(x) =$

- ☐ 1
- ☐ $x - 1$
- ☐ $\sqrt{\sqrt{x}}$
- ☐ $\sqrt{\sqrt{x} - 1}$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned} 2x - 3 &< -7 \qquad \text{add 3} \\ 2x &< -7 + 3 \\ 2x &< -4 \qquad \text{divide by 2} \\ x &< -2 \end{aligned}$$

Hence the solution is $(-\infty, -2)$. ■

Solution to 2.

$$
\begin{aligned}
-2 < 2-2x &< 3 && \text{subtract } 2\\
-4 < \quad -2x &< 1 && \text{divide by } -2 \text{ and switch the inequality}\\
& && \text{because we are dividing by negative number}\\
2 > \qquad x &> \frac{-1}{2} && \text{rewrite the inequality}\\
\frac{-1}{2} < \qquad x &< 2
\end{aligned}
$$

Hence the solution is $(-1/2, 2)$. ■

Solution to 3.

First, we write	$x^2 + 4x + 3 < 0$	we are looking for $(-)$ sign.
Second, set	$x^2 + 4x + 3 = 0$	to find the zeroes.
We have	$(x+3)(x+1) = 0$	by factor.
Hence	$x = -3, -1.$	

Third, use the real line to find the sign of $x + 3$ and $x + 1$.

	$-\infty$ to -3	-3 to -1	-1 to ∞
Test Value	-4	-2	1
sign of $x+3$	$-4+3=-1 \quad (-)$	$-2+3=1 \quad (+)$	$1+3=4 \quad (+)$
sign of $x+1$	$-4+1=-3 \quad (-)$	$-2+1=-1 \quad (-)$	$1+1=2 \quad (+)$
sign of $(x+3)(x+1)$	$(-)\cdot(-)=+$	$(+)\cdot(-)=-$	$(+)\cdot(+)=+$

Hence the solution is $(-3, -1)$. ■

Solution to 4.

$$
\begin{aligned}
|2x+1| < 1 &\Leftrightarrow -1 < 2x+1 < 1 && \text{Use } |x| < k \Leftrightarrow -k < x < k. \\
&\Leftrightarrow -2 < 2x < 0 && \text{subtract 1 from all sides} \\
&\Leftrightarrow -1 < x < 0 && \text{divide all sides by 2}
\end{aligned}
$$

Hence the interval is $(-1, 0)$. ■

Solution to 5.

$$\begin{aligned} |2x+1| > 2 &\Leftrightarrow 2x+1 > 2 \quad \text{or} \quad 2x+1 < -2. \\ &\Leftrightarrow 2x > 1 \quad \text{or} \quad 2x < -3 \\ &\Leftrightarrow x > \frac{1}{2} \quad \text{or} \quad x < \frac{-3}{2} \end{aligned}$$

Hence the interval is $(-\infty, -3/2) \cup (1/2, \infty)$. ■

Solution to 6. Since $\dfrac{x+2}{x-2} \geq 0$, then we are looking for $(+)$ sign. Next, we find the zeros of the numerator and the denominator. The real zeros of the numerator and the denominator are $x+2=0 \Leftrightarrow x=-2$ and $x-2=0 \Leftrightarrow x=2$. So the expression's test intervals are $(-\infty,-2], [-2,2)$, and $(2,\infty)$. We excluded -2 because we have bigger than sign and excluded 2 because it makes the denominator equal zero and dividing by zero is not allowed. Now, we use the real line to find the sign of $x+2$ and $x-2$.

$-\infty$	-2	2	∞
Test Value	-3	0	3
sign of $x+2$	$-3+2=-1 \quad (-)$	$0+2=2 \quad (+)$	$3+2=5 \quad (+)$
sign of $x-2$	$-3-2=-5 \quad (-)$	$0-2=-2 \quad (-)$	$3-2=1 \quad (+)$
sign of $(x+2)/(x-2)$	$(-)/(-)=+$	$(+)/(-)=-$	$(+)/(+)=+$

We find the $(+)$signs in the interval $(-\infty,-2]$ or $(2,\infty)$. Hence the solution is $(-\infty,-2] \cup (2,\infty)$. ■

Solution to 7.

$$\begin{aligned} d((5,2),(1,-1)) &= \sqrt{(5-1)^2+(2-(-1))^2} \\ &= \sqrt{(4)^2+(3)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

■

Solution to 8.

$$\begin{aligned} d((1,1),(3,4)) &= \sqrt{(1-3)^2+(1-4)^2} \\ &= \sqrt{(-2)^2+(-3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}. \end{aligned}$$

$$\begin{aligned} d((1,1),(0,6)) &= \sqrt{(1-0)^2+(1-6)^2} \\ &= \sqrt{(1)^2+(-5)^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26}. \end{aligned}$$

$$\begin{aligned} d((3,4),(0,6)) &= \sqrt{(3-0)^2+(4-6)^2} \\ &= \sqrt{(3)^2+(-2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13}. \end{aligned}$$

Now, since $(\sqrt{13})^2 + (\sqrt{13})^2 = 13 + 13 = 26 = (\sqrt{26})^2$, we have a right triangle. ■

Solution to 9. Yes. The slope of the line joining the points $(2, 1), (0, 2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 1}{0 - 2} = -\frac{1}{2},$$

while the slope of the line joining the points $(0, 2), (4, 0)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{0 - 2}{4 - 0} = \frac{-2}{4} = -\frac{1}{2}.$$

■

Solution to 10. The slope of the line through the points $(3, -6)$ and $(1, -1)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{-1 - (-6)}{1 - 3} = \frac{5}{-2} = -\frac{5}{2}.$$

■

Solution to 11. Since $m = \frac{\Delta y}{\Delta x} = \frac{-1}{2}$, this mean that to get another point from the point $(1, 2)$ just add (or subtract) $\Delta x = 2$ to the x-coordinate of the point $(1, 2)$ and add (or subtract) $\Delta y = -1$ to the y-coordinate of the point $(1, 2)$. Hence $(1 + 2, 2 + (-1)) = (3, 1)$ is a point on the line. ■

Solution to 12.

$$\begin{aligned} y - 3x - 1 &= 0 & 9y + 3x &= -6 \\ y &= 3x + 1 & 9y &= -3x - 6 \\ & & y &= \frac{-1}{3}x - 3 \\ m_1 &= 3 & m_2 &= \frac{-1}{3} \end{aligned}$$

Hence the lines are perpendicular. ■

Solution to 13. we find the slope of the line $y - 2x = 1$

$$y - 2x = 1 \qquad \text{Isolate } y \text{ term.}$$
$$y = 2x + 1 \qquad \text{Point-Slope form}$$

Now, since the line is perpendicular to $y - 2x = 1$ then $m = \frac{-1}{2}$ and passes through the point $(3, 1)$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-Slope form}$$
$$y - 1 = -\frac{1}{2}(x - 3) \qquad \text{Substitute}$$
$$y - 1 = -\frac{1}{2}x + \frac{3}{2} \qquad \text{Simplify}$$
$$y = -\frac{1}{2}x + \frac{3}{2} + 1 \qquad \text{Simplify}$$
$$Y = -\frac{1}{2}x + \frac{5}{2}$$

■

Solution to 14. The function $f(x) = f(x) = x - 1$ is a polynomial, hence

$$D(f) = \mathbb{R}.$$

■

Solution to 15. $f(x) = \sqrt{2x{-}3}$ is even root function. Then it is defined if

$$\begin{aligned} 2x-3 &\geq 0 \\ 2x &\geq 3 \\ x &\geq 3/2 \end{aligned}$$

Hence $D(f) = [3/2, \infty)$. ■

Solution to 16. The function is a rational function. The domain is $\mathbb{R} \setminus \{$ zeros of $x^2+4\}$. Now, if $x^2+4=0 \Leftrightarrow x^2 = -4$, but this is impossible because the square of any real number is bigger than or equal to zero and never negative. Hence $x^2+4 \neq 0$. Hence $D(f) = \mathbb{R}$. ∎

Solution to 17. The function $f(x) = \sqrt[3]{x^2 - 1}$ is an odd function and hence $D(f) = \mathbb{R}$. ■

Solution to 18. The function is a rational function. The domain is $\mathbb{R} \setminus \{$ zeros of $x^2 - x - 6\}$.

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x - 3 = 0 \text{ or } x + 2 &= 0. \\ x = 3 \text{ or } \qquad x &= -2. \end{aligned}$$

Hence $D(f) = \mathbb{R} \setminus \{-2, 3\} = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

■

Solution to 19. To find the x-intercepts, set $f(x) = 0$.

$$\begin{aligned} \frac{2x-1}{x^2-4} = 0 \Leftrightarrow 2x - 1 &= 0. \\ \Leftrightarrow 2x &= 1 \\ \Leftrightarrow x &= 1/2. \end{aligned}$$

To find the y-intercept, set $x = 0$. $f(0) = \dfrac{2(0)-1}{(0)^2-4} = \dfrac{1}{4}$. ■

Solution to 20. We can see that $x^2 - 2x - 8 = 0 \Leftrightarrow (x-4)(x+2) = 0 \Leftrightarrow x = 4, x = -2$. We also can use the quadratic formula to find all the zeros of $f(x) = x^2 - 2x - 8$. Here $a = 1$, $b = -2$, and $c = -8$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4 + 32}}{2} \\
&= \frac{2 \pm 6}{2} \\
x &= \frac{2-6}{2} = -2 \quad \text{or } x = \frac{2+6}{2} = 4.
\end{aligned}$$

■

Solution to 21. To determine the points of intersection of the two graphs, we set the two functions equal and solve for x :

$$\begin{aligned} x^2 - 2x + 4 &= 3x - 2 \\ x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0 \\ x - 3 = 0 \text{ or } &\quad x - 2 = 0 \\ x = 3 \text{ or } &\quad x = 2. \end{aligned}$$

Now, if $x = 2 \Rightarrow y = 3(2) - 2 = 6 - 2 = 4$, and if $x = 3 \Rightarrow y = 3(3) - 2 = 9 - 2 = 7$. Hence the points of intersection are $(2, 4), (3, 7)$. ■

Solution to 22.

$$\begin{aligned}120^\circ &= \cancel{120}^{2} \cdot \frac{\pi}{\cancel{180}^{3}} \\ &= \frac{2\pi}{3}\end{aligned}$$

■

Solution to 23.

$$\frac{4\pi}{3} = \frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}}$$
$$= 4 \cdot 60 = 240^\circ$$

■

Solution to 24. We see from the table below that

$$\sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}.$$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

■

Solution to 25.

$$\begin{aligned}\sin\left(\frac{5\pi}{3}\right) &= \sin\left(2\pi - \frac{\pi}{3}\right)\\ &= \sin(2\pi)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)\cos(2\pi)\\ &= (0)(\frac{1}{2}) - (\frac{\sqrt{3}}{2})(1)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

■

Solution to 26.

$$\begin{aligned}
\cos(2x) &= \cos(x+x) && \text{use } \cos(x+y) = \cos x\cos y - \sin y\sin x \\
&= \cos x\cos x - \sin x\sin x && \cos x\cdot\cos x = \cos^2 x \text{ and } \sin x\cdot\sin x = \sin^2 x, \\
&= \cos^2 x - \sin^2 x && \cos^2 x + \sin^2 x = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x \\
\cos(2x) &= \cos^2 x - (1 - \cos^2 x) && \\
\cos(2x) &= \cos^2 x - 1 + \cos^2 x && \text{add and solve for } \cos^2 x \\
\cos(2x) &= 2\cos^2 x - 1 && \text{add and solve for } \cos^2 x \\
2\cos^2 x &= 1 + \cos(2x) && \text{divide by 2} \\
\cos^2 x &= \frac{1+\cos(2x)}{2} &&
\end{aligned}$$

■

Solution to 27.

$$\begin{aligned}\cos(x+\pi) &= \cos x\cos(\pi) - \sin x\sin(\pi) && \cos(x+y) = \cos x\cos y - \sin y\sin x,\\ &= \cos x\cdot(-1) - \sin x\cdot 0 && \cos(\pi) = -1, \sin(\pi) = 0\\ &= -\cos x\end{aligned}$$

■

Solution to 28.

$$\begin{aligned}(g-f)(x) &= g(x)-f(x) && \text{by definition.}\\ &= \sqrt{x-1}-(x^2+1)\\ &= \sqrt{x-1}-x^2-1.\end{aligned}$$

■

Solution to 29. The function $f(x) = \sqrt{x-3}$ is an even root function, hence f is defined if

$$x - 3 \geq 0 \Leftrightarrow x \geq 3.$$

Hence $D(f) = [3, \infty)$. The function $g(x) = x - 5$, is a polynomial, hence $D(g) = \mathbb{R}$. Now,

$$\begin{aligned} D(f/g) &= (D(f) \cap D(g)) \setminus \{x : g(x) = 0\} \\ &= (\mathbb{R} \cap [3, \infty)) \setminus \{x : x - 5 = 0\} \\ &= [3, \infty) \setminus \{x : x - 3 = 0\} = [3, \infty) \setminus \{5\} \\ &= [3, 5) \cup (5, \infty). \end{aligned}$$

■

Solution to 30. The function $f(x)=\sqrt{25-x^2}-\sqrt{x^2-4}$ is the difference of $\sqrt{25-x^2}$ and $\sqrt{x^2-4}$.

Hence $D(f)=D(\sqrt{25-x^2})\cap D(\sqrt{x^2-4})$.

The function $\sqrt{25-x^2}$ is an even root function, then

$$
\begin{aligned}
25-x^2\geq 0 &\Leftrightarrow x^2\leq 25 && \text{move } x^2 \text{ to the other side}\\
&\Leftrightarrow \sqrt{x^2}\leq 5 && \text{take the square root}\\
&\Leftrightarrow |x|\leq 5 && \sqrt{x^2}=|x| \text{ use properties of}\\
&\Leftrightarrow -5\leq x\leq 5 && \text{absolute value inequality}
\end{aligned}
$$

Hence $D(\sqrt{25-x^2})=[-5,5]$.

The function $\sqrt{x^2-4}$ is an even root function, then

$$
\begin{aligned}
x^2-4\geq 0 &\Leftrightarrow \sqrt{x^2}\geq 2 && \text{take the square root}\\
&\Leftrightarrow |x|\geq 2 && \sqrt{x^2}=|x| \text{ use properties of}\\
&\Leftrightarrow x\geq 2 \text{ or } x\leq -2 && \text{absolute value inequality}
\end{aligned}
$$

Hence $D(\sqrt{x^2-4})=(-\infty,-2]\cup[2,\infty)$.

$$\begin{aligned} D(f) &= D(\sqrt{25-x^2}) \cap D(\sqrt{x^2-4}) \\ &= [-5,5] \cap (-\infty,-2] \cup [2,\infty) \\ &= [-5,-2] \cup [2,5]. \end{aligned}$$

■

Solution to 31.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{x+3}) \\ &= (\sqrt{x+3})^2 - 1 \\ &= x + 3 - 1 = x + 2.\end{aligned}$$

■

Solution to 32. $D(f \circ g) = D(g) \cap D(f(g(x)))$. Since $(f \circ g)(x) = f(g(x)) = x + 2$, then $D(g(f(x))) = \mathbb{R}$. Since $g(x) = \sqrt{x+3}$, is an even root function,
then $x + 3 \geq 0 \Leftrightarrow x \geq -3$. Hence $D(f) = [-3, \infty)$.

$$\begin{aligned} D(f \circ g) &= D(g) \cap D(f(g(x))) \\ &= [-3, \infty) \cap \mathbb{R} \\ &= [-3, \infty). \end{aligned}$$

■

Solution to 33.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{\sqrt{x} - 1}.\end{aligned}$$

■