

THE FINAL EXAMINATION OF PHYSICS 454, 16/11/1423

Q1

Use the Hellmann-Feynman theorem to obtain $\langle \frac{1}{r^2} \rangle$ for the hydrogen atom. Where the Hamiltonian is

$$\hat{H} = -\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}\right) + \frac{l(l+1)}{2r} - \frac{1}{r} \quad \text{and} \quad E = -\frac{1}{2n^2}, n = k+l+1$$

b- Which of the following potentials can be considered as solutions of the Thomas-Fermi equation and why?

$$\Phi(x) = 1 - \alpha x + \beta x^{3/2} + \dots, \Phi(x) = axe^{-\alpha x}, \quad _, \Phi(x) = (ae^{-\alpha x} + be^{-\beta x})$$

Q2

Calculate the energies of the harmonic oscillator to the second order, whose Hamiltonian is $H = H_0 + ae^{-bx^2}$.

The wave functions are $\Psi_0 = Ae^{-ax^2}$, $\Psi_1 = Bxe^{-bx^2}$

Q3

Given the matrix

$$\begin{pmatrix} 1 & i & 1 \\ -i & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

- a- Obtain its cofactor and its inverse.
- b- What are the eigenvalue and eigenvector?

Q4

Given the scattering amplitude in the form

$$f(\Theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1)e^{i\delta_l} \sin \delta_l P_l(\cos \Theta)$$

What is the total scattering cross section? Show that $\sigma = \frac{4\pi}{k} \text{Im}[f(0)]$

b-For a finite square well the potential is $V(r) = \begin{cases} -V_0 & , r < a \\ 0 & , r > a \end{cases}$. Solve the Schrodinger equation inside and outside the well to obtain the total cross section ($l = 0$).

Q5-

Use the Born approximation to find the differential cross section and the total cross section when a projectile

is scattered by a potential of the form $V(r) = -\frac{V_0}{r_0}(r+r_0), 0 \leq r \leq r_0$.

Q6-

A system is under a potential of the form $V(x) = \gamma|x|^{3/2}, \gamma > 0$.

Use the WKB method to obtain the energy of the system.

Q7- Write names of four books you have used during this course.

Hint: $1 - \int_0^{\infty} e^{-bx^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{b}}$,

$$\int_0^1 t^{y-1} (1-t)^{w-1} dt = \frac{\Gamma(y)\Gamma(w)}{\Gamma(y+w)}$$