

THE FINAL EXAMINATION OF PHYSICS 342, 16/11/1423

TIME: TWO HOURS

Q1

a- In the photoelectric experiment, the kinetic energy of the electron is 2.2eV for radiation of 200 nm and

1.0eV for radiation of 248 nm. Use these data to calculate the cutoff wavelength and the Planck's constant

b- A black cavity is placed in an oven of a temperature of 1650K. What is the value of the maximum

wavelength and what rate does energy escape from the cavity through a hole in its surface of a diameter of

1.0mm?

Q2

a- Two traveling waves with $\lambda_1 = 1\text{m/s}$ and $\lambda_2 = 10/9\text{m/s}$, $v_1 = 3\text{m/s}$ and $v_2 = 2.5\text{m/s}$. What are the group velocity, v_g , and the phase velocity, v_p , of their superposition (hint: $v = \frac{\omega}{k}$)?

b- Show that $V_g = v_p - \lambda \frac{dv_p}{d\lambda}$

Q3

Use the Schrodinger equation for the harmonic oscillator with the wave function $\Psi = axe^{-ax^2}$ to find the

parameter a and the energy, E , of the harmonic oscillator.

Q4

- a- Given the wave function $\Psi(x) = A \sin \frac{n\pi x}{L}$, $0 \leq x \leq L$. Obtain the value of the constant A, what we call it ?
- b- What is the probability of finding the particle in the region $0 \leq x \leq \frac{L}{4}$?
- c- In the Thomson model, derive the deflection angle and show that $\theta_{\max} = \frac{zkR^2}{mv^2}$, z is the α - particle charge

$$\text{and } k = \frac{Ze^2}{4\pi\epsilon_0 R^3}.$$

Q5

- a- The angular momentum of the electron in the hydrogen atom is given by $2.15 \times 10^{-34} \text{ J}\cdot\text{s}$. Find its velocity and the orbital radius.
- b- What angles does \hat{L} vector make with the Z-axis when $l=2$. Compare the angular momentum in this case with that of Bohr.
- c- Define and derive the Zeeman effect.
- d- Compute the change in the wavelength of the $3p \rightarrow 2s$ photon when a hydrogen atom is placed in a magnetic field of 4T.

$$h = 6.625 \times 10^{-34} \text{ J}\cdot\text{s}, e = 1.602 \times 10^{-19} \text{ C}, m = 9.1 \times 10^{-31} \text{ kg}$$

Q1

Use the Hellmann-Feynman theorem to obtain $\langle \frac{1}{r^2} \rangle$ for the hydrogen atom. Where the Hamiltonian is

$$\hat{H} = -\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}\right) + \frac{l(l+1)}{2r} - \frac{1}{r} \quad \text{and} \quad E = -\frac{1}{2n^2}, n = k+l+1$$

b- Which of the following potentials can be considered as solutions of the Thomas-Fermi equation and why?

$$\Phi(x) = 1 - \alpha x + \beta x^{3/2} + \dots, \Phi(x) = \alpha x e^{-\alpha x}, \dots, \Phi(x) = (a e^{-\alpha x} + b e^{-\beta x})$$

Q2

Calculate the energies of the harmonic oscillator to the second order, whose Hamiltonian is $H = H_0 + a e^{-\beta x^2}$.

The wave functions are $\Psi_0 = A e^{-\alpha x^2}$, $\Psi_1 = B x e^{-\alpha x^2}$

Q3

Given the matrix

$$\begin{pmatrix} 1 & i & 1 \\ -i & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

- a- Obtain its cofactor and its inverse.
 - b- What are the eigenvalue and eigenvector?
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Q4

Given the scattering amplitude in the form

$$f(\Theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \Theta)$$

What is the total scattering cross section? Show that $\sigma = \frac{4\pi}{k^2} \text{Im}[f(0)]$

b-For a finite square well the potential is $V(r) = \begin{cases} -V_0 & , r < a \\ 0 & , r > a \end{cases}$. Solve the Schrodinger equation inside and outside the well to obtain the total cross section ($l = 0$).

Q5-

Use the Born approximation to find the differential cross section and the total cross section when a projectile

is scattered by a potential of the form $V(r) = -\frac{V_0}{r_0}(r+r_0), 0 \leq r \leq r_0$.

Q6-

A system is under a potential of the form $V(x) = \gamma|x|^{3/2}, \gamma > 0$.

Use the WKB method to obtain the energy of the system.

Q7- Write names of four books you have used during this course.

$$\text{Hint: } 1 - \int_0^{\infty} e^{-bx^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{b}},$$

$$\int_0^1 t^{y-1} (1-t)^{w-1} dt = \frac{\Gamma(y)\Gamma(w)}{\Gamma(y+w)}$$

THE FINAL EXAMINATION OF PHYSICS 353, 7/4/1424H

TIME: TWO HOURS

Q1

Show that $\int_{-\infty}^{\infty} f(y)\delta(y)dy = f(0)$ and $\int_{-\infty}^{\infty} \delta(y)dy = 1$ and $\delta(ay) = \frac{\delta(y)}{a}$, a is a constant

Q2

Use any learned method to obtain an explicit form of $\Psi_{321}(r, \theta, \varphi) = R_{32}(r)Y_2^1(\theta, \varphi)$

What is the energy of an electron in this state? What is its angular momentum?

Q3

The energy eigenvalues of a simple harmonic oscillator is given by

$$E_n = \hbar \omega_0 \left(n + \frac{1}{2} \right), n = 1, 2, 3, \dots$$

Show that $n \geq -\frac{1}{2}$.

The potential of a simple harmonic oscillator is $V(x) = \frac{1}{2} kx^2$. Show that

$$\langle V \rangle = \frac{\hbar \omega_0}{2} \left(n + \frac{1}{2} \right).$$

Show that $\alpha^+ |n\rangle = \sqrt{n+1} |n+1\rangle$

Q4

Use the raising operator $L^+ = \hbar e^{i\phi} \left(\frac{\partial}{\partial \theta} + i \cot \theta \frac{\partial}{\partial \phi} \right)$ to obtain the spherical harmonics

$$Y_2^2(\theta, \phi).$$

Q5

If $\frac{d \langle A \rangle}{dt} = \frac{i}{\hbar} \langle \Psi | [\hat{H}, \hat{A}] | \Psi \rangle$, Show that $\frac{d \langle x \rangle}{dt} = \bar{v}$, where \bar{v} is the average velocity of a particle.

Q6

Show that for a wave function of the form $\Psi(x, t) = A e^{i\phi(x, t)}$, the current density takes the form

$$J = \frac{\hbar}{m} |A|^2 \frac{\partial f}{\partial x}$$

Q7

For the step potential with $V = 2E$, find the transmission and the reflection coefficients of a traveling

wave of energy E . Show that $T+R=1$

Q8

For the hydrogen atom, take the Hamiltonian of the form

$$H = (p^2 c^2 + m^2 c^4)^{1/2} - mc^2 + V, \quad V = -\frac{Ze^2}{r}, \text{ and}$$

Obtain its total energy.